Hecke group algebras as degenerate affine Hecke algebras

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arXiv:0804.3781v1 [math.RT]

It all started at FPSAC 2006, San Diego

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Thanks Mike!

Coxeter groups

Definition (Coxeter group W)

Generators : $(s_i)_{i\in S}$ (simple reflections)

Relations: $s_i^2 = 1$ and $\underbrace{s_i s_j \cdots}_{m_{i,j}} = \underbrace{s_j s_i \cdots}_{m_{i,j}}$, for $i \neq j$

Group algebra: $\mathbb{C}[W]$

Example (Type A_n: symmetric group $\mathfrak{S}_{n+1}$)

Generators: $(s_i)_{i=1,\dots,n}$ (elementary transpositions)

Relations:

$$
\begin{aligned}
s_i^2 &= 1 && \text{for all } 1 \leq i \leq n,\\
s_i s_j &= s_j s_i && \text{for all } |i-j| > 1,\\
s_i s_{i+1} s_i &= s_{i+1} s_i s_{i+1} && \text{for all } 1 \leq i \leq n-1.
\end{aligned}
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0-(Iwahori)-Hecke algebras

Definition (0-Hecke algebra H(W)(0))

Generators : $(\pi_i)_{i\in S}$

Relations: $\pi_i^2 = \pi_i$ and $\underbrace{\pi_i\pi_j\cdots}_{m_{i,j}} = \underbrace{\pi_j\pi_i\cdots}_{m_{i,j}}$ for $i \neq j$

Basis: $(\pi_w)_{w\in W}$

Example (Type A_n)

Generators: $(\pi_i)_{i=1,\dots,n}$ (adjacent comparators)

Relations:

$$\begin{aligned} \pi_i^2 &= \pi_i && \text{for all } 1 \le i \le n,\\ \pi_i\pi_j &= \pi_j\pi_i && \text{for all } |i-j|>1,\\ \pi_i\pi_{i+1}\pi_i &= \pi_{i+1}\pi_i\pi_{i+1} && \text{for all } 1 \le i \le n-1. \end{aligned}$$

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\end{aligned}$$

0-Hecke algebras and bubble sort

Example (Right regular representation for type A)

13854627	13854627	13854627
↓ s_2 ↓ s_6	↓ π_2 ↓ π_6	↓ $\overline{\pi}_2$ ↓ $\overline{\pi}_6$
18354267	18354627	13854267
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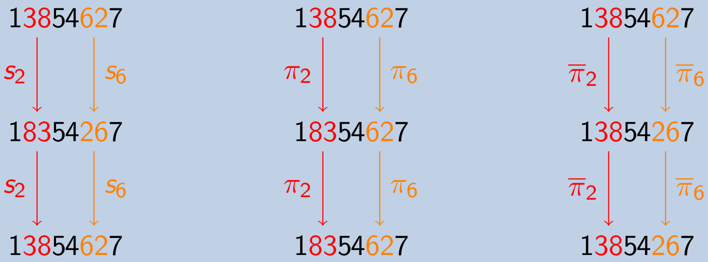

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Example (Bubble sort)

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0-Hecke algebras and bubble sort

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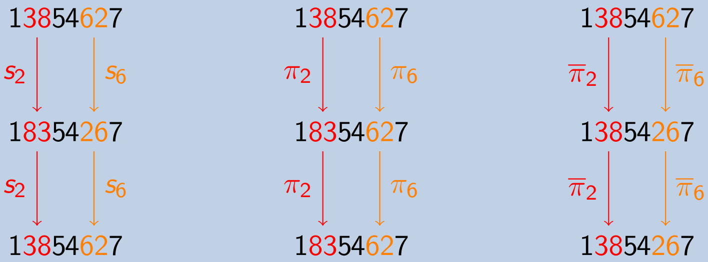

Example (Bubble sort)

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0-Hecke algebras and bubble sort

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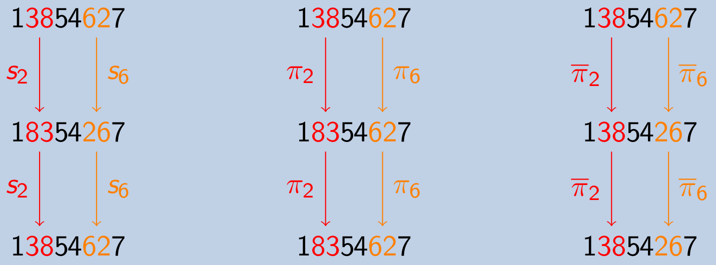
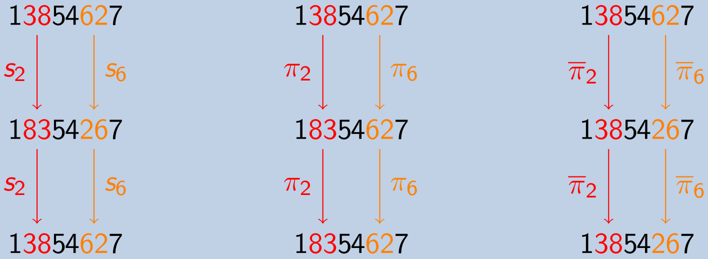


Example (Bubble sort)

81354762

0-Hecke algebras and bubble sort

Example (Right regular representation for type A)

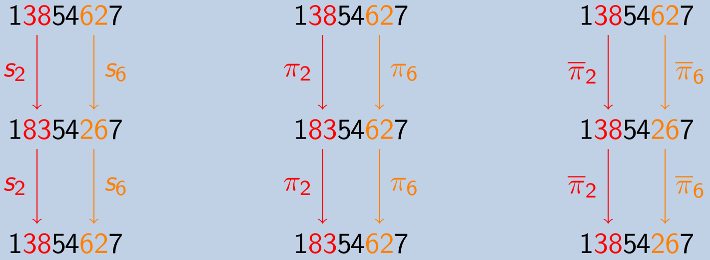


Example (Bubble sort)

81357462

0-Hecke algebras and bubble sort

Example (Right regular representation for type A)



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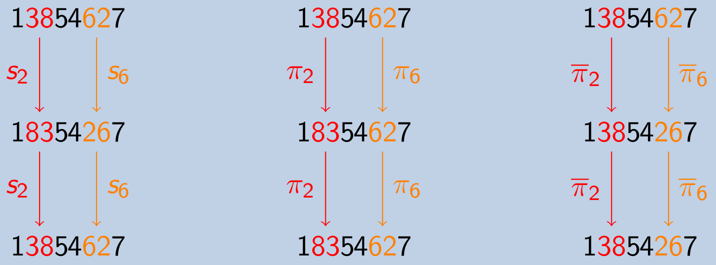

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Example (Bubble sort)

87136542

0-Hecke algebras and bubble sort

Example (Right regular representation for type A)

13854627	13854627	13854627
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13854627	18354627	13854267

Example (Bubble sort)

87163542

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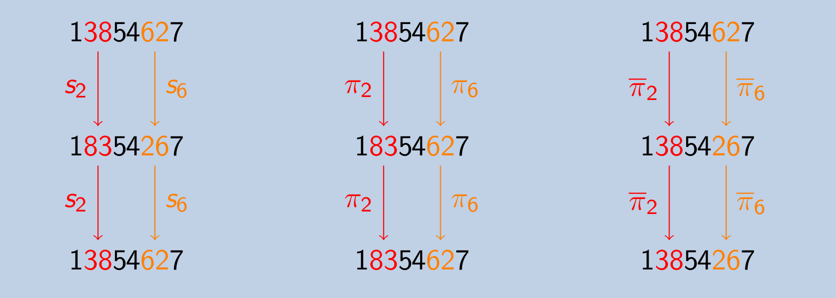

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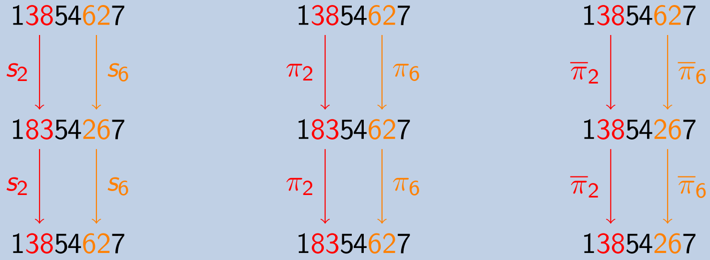

Example (Bubble sort)

87651342

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Example (Bubble sort)

87651432

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Example (Bubble sort)

87654132

0-Hecke algebras and bubble sort

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13854627	13854627	13854627
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18354267	18354627	13854267
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Example (Bubble sort)

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Example (Bubble sort)

87654312

0-Hecke algebras and bubble sort

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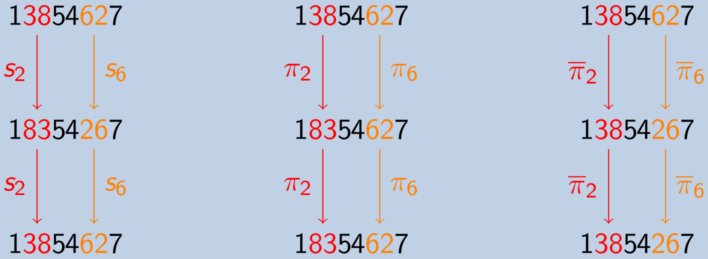

Example (Bubble sort)

87654321

Hecke algebras

Take q_1 and q_2 parameters, and set $q := -\frac{q_1}{q_2}$.

Definition (Hecke algebra $\mathrm{H}(W)(q_1, q_2)$)

Generators : $(T_i)_{i\in S}$ Relations: $(T_i - q_1)(T_i - q_2) = 0$ and
$\underbrace{T_i T_j \cdots}_{m_{i,j}} = \underbrace{T_j T_i \cdots}_{m_{i,j}}$, for $i \neq j$

Basis: $(T_w)_{w\in W}$

- At $q = 1$: group algebra $\mathbb{C}[W]$
- At $q = 0$: 0-Hecke algebra $\mathrm{H}(W)(0)$
- At q not 0 nor a root of unity: isomorphic to $\mathbb{C}[W]$

Realization of T_i as operator in $\mathrm{End}(\mathbb{C}W)$:

$$T_i := (q_1 + q_2)\pi_i - q_1 s_i$$

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A silly idea during a brainstorm (Thibon, Novelli, H., T., 2003)

Definition (Hecke group algebra HW of a Coxeter group W)

Glue $\mathbb{C}[W]$ and H$(W)(0)$ on their right regular representations:

$$HW := \langle (\pi_i, s_i)_{i \in S} \rangle \quad \subset \quad \mathrm{End}(\mathbb{C}W)$$

- Any interesting structure?
- Contains all Hecke algebras by construction
- Type A: dimension and dimension of the radical in the Sloane!

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The Hecke group algebra of rank 1

$$W := \{1, s\} \qquad \mathbb{C}W := \mathbb{C}.1 \oplus \mathbb{C}.s$$

$$\mathrm{id} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad s = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \pi = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad \overline{\pi} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Relations and some natural bases of HW

$$s\pi = \pi, \qquad \pi s = \overline{\pi}, \qquad \overline{\pi} + \pi = 1 + s$$

$$\{\mathrm{id}, s, \pi\} \qquad \text{or} \qquad \{\mathrm{id}, \pi, \overline{\pi}\}$$

Dimension 1 simple and projective modules

$$(1-s).\mathrm{id} = (1-s), \qquad (1-s).s = -(1-s), \qquad (1-s).\pi = 0$$

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Relations and some natural bases of $\mathrm{H}W$

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Dimension 1 simple and projective modules

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Structure of Hecke group algebras

Theorem (H.,T., 2005)

- HW *algebra of left antisymmetry preserving operators*
- HW^* *algebra of left symmetry preserving operators*
- *Basis of* HW*:* $\{w\pi_{w'} \mid D_R(w) \cap D_L(w') = \emptyset\}$
- *Rep. theory governed by the combinatorics of descents*
- HW *Morita equivalent to the poset algebra of boolean lattice*
- *Projective & simple modules indexed by parabolic subgroups*
  Restriction of simple:
  - *Exactly the Young's ribbon representation of* W
  - *Exactly the projective modules of* $H(W)(0)$

Question (Thibon 2005)

Is there a link with the affine Hecke algebra?

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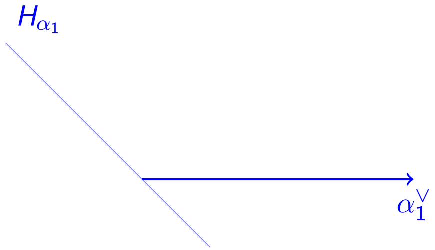

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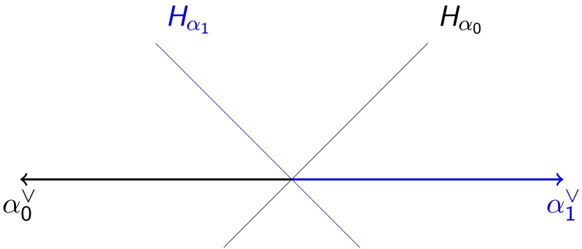

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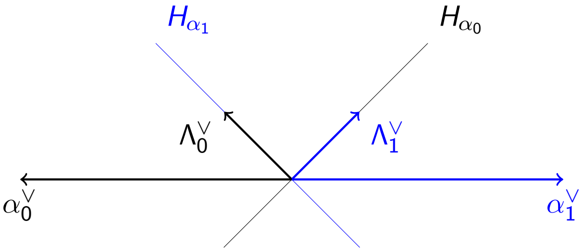

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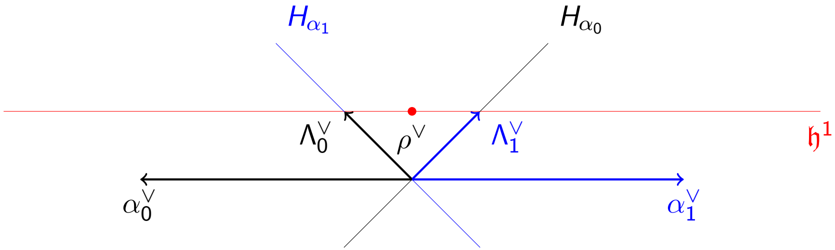

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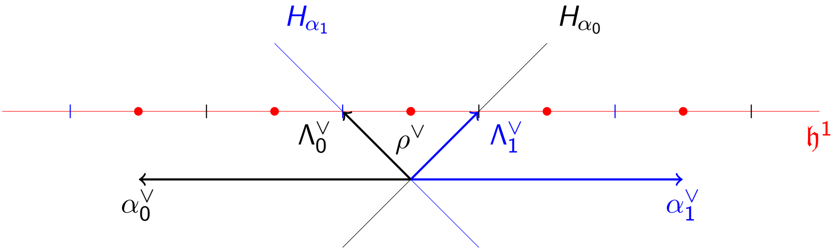

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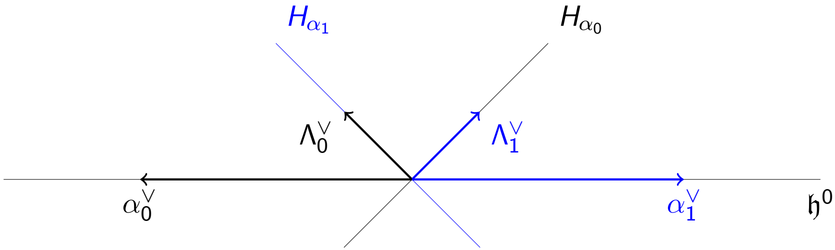

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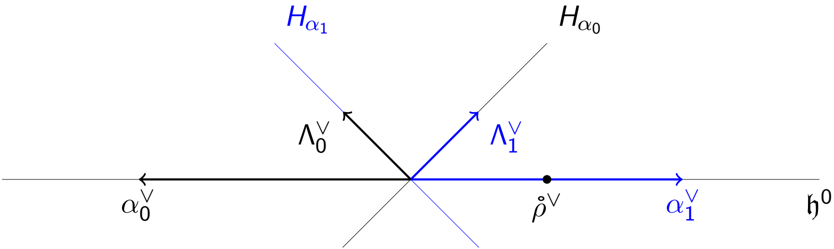

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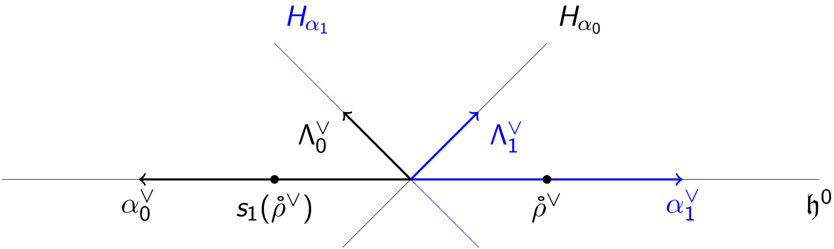

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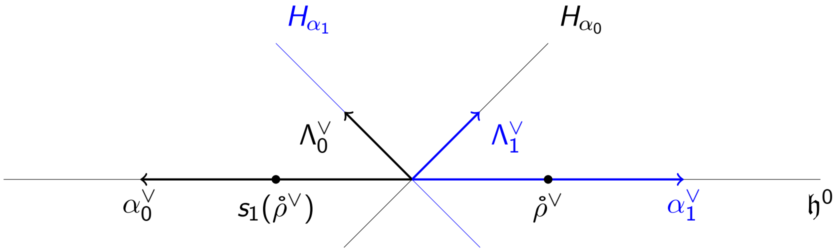

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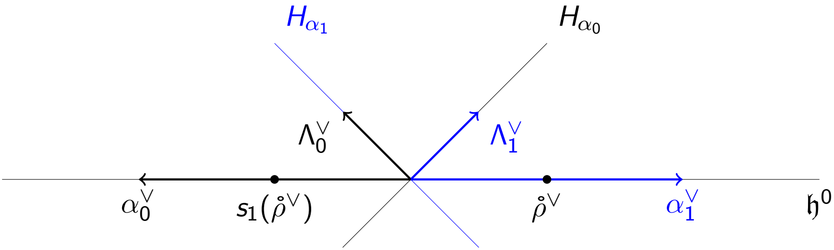

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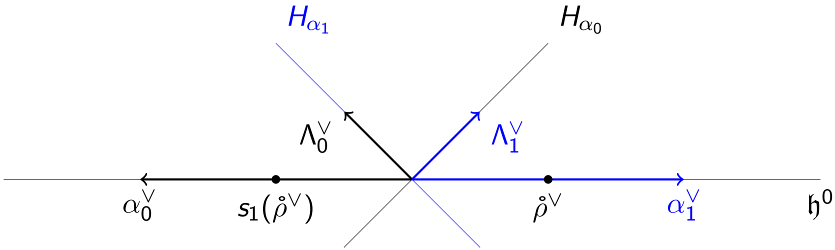

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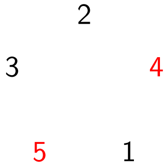

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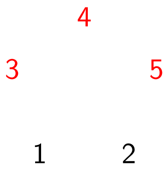

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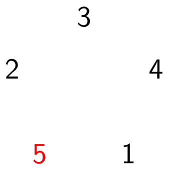

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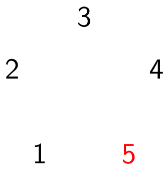
$\pi_0, \pi_1, \ldots, \pi_n$ act transitively on permutations

Recursive sorting algorithm in type A

$\pi_1, \ldots, \pi_n$ can antisort 12345 to 54321 (bubble sort)
But not back! (going down the permutohedron)

Definition (Affine action)

Put the permutation on a circle
π_0 acts between last and first letter



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2 5

1 4

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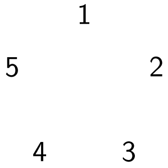

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4 3

Proposition

$\pi_0, \pi_1, \ldots, \pi_n$ *act transitively on permutations*

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

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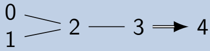
- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{1234}$$

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$$21\underline{34}$$

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Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$2\underline{3}1\underline{4}$$

- Type-free induction strategy
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- Type B:

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$2\underline{34}1$

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$$\underline{3}2\underline{4}1$$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
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Proof

- Type B:

$$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$$

$$\underline{34}21$$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
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- Type B: 

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  $\underline{34}21$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

4321

- Type-free induction strategy
- Case by case induction step
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Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$432\underline{1}$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$43\underline{1}2$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$$

$$4\underline{1}32$$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$$

$\underline{1}432$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{1}43\color{red}{2}$

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$\underline{1}43\underline{2}$

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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{1}4\underline{2}3$

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Recursive sorting algorithms in other types

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- *Existence for all types (including twisted)*

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- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{1}\underline{2}43$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{12}43$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: $0, 1 > 2 - 3 \Rightarrow 4$ $\qquad 1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{12}4\underline{3}$$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{123}4$$

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21$\underline{3}$4

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$$21\underline{3}4$$

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- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$2\underline{3}14$$

- Type-free induction strategy
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$2\underline{3}14$

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Proposition (S.,T.,2007)

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Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}214$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: $\begin{smallmatrix}0\\1\end{smallmatrix} > 2 — 3 \Rightarrow 4$ $\qquad 1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}214$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}241$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}24\underline{1}$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{3}2\underline{1}4$$

- Type-free induction strategy
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}\underline{1}24$

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$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{31}24$

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1324

- Type-free induction strategy
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- Type B:

$$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$$

1342

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$134\underline{2}$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

13$\underline{2}$4

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

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- *Similar algorithms for types B, C, D*
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Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

1$\underline{2}$34

- Type-free induction strategy
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Recursive sorting algorithms in other types

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- Type B:

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$$\underline{2}134$$

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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{2}314$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{2}341$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{2}34\underline{1}$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{2}3\underline{1}4$

- Type-free induction strategy
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

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- Type B: 

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$\underline{2}\,\underline{1}34$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

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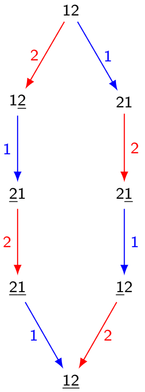
Type free geometric argument (I)



π_1, π_2 on C_2

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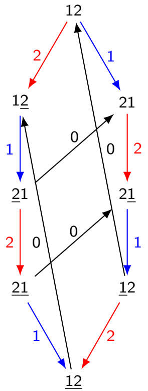

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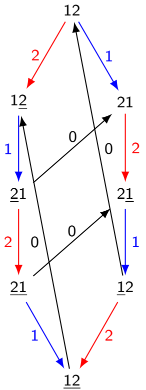

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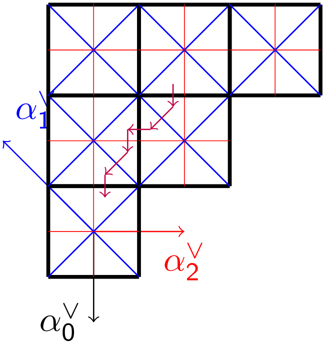

Alcove picture at level 1

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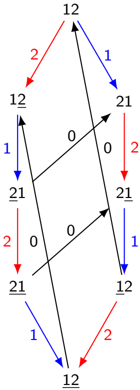

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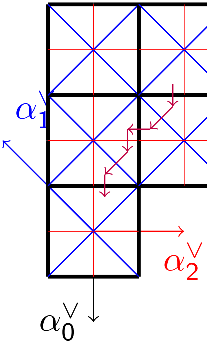

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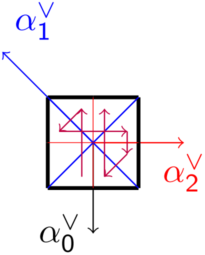
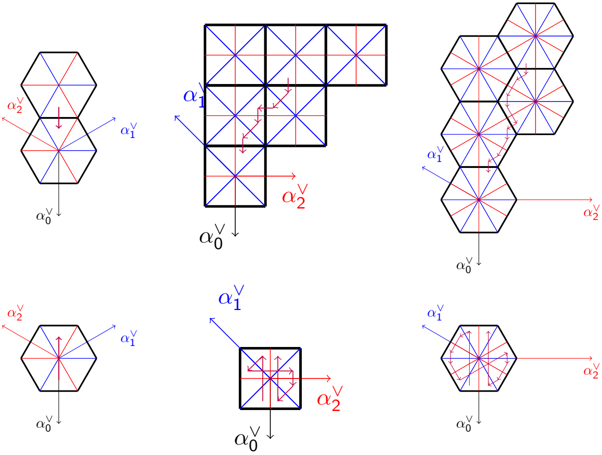


Quotient at level 0
(Steinberg torus)

Type free geometric argument (II)



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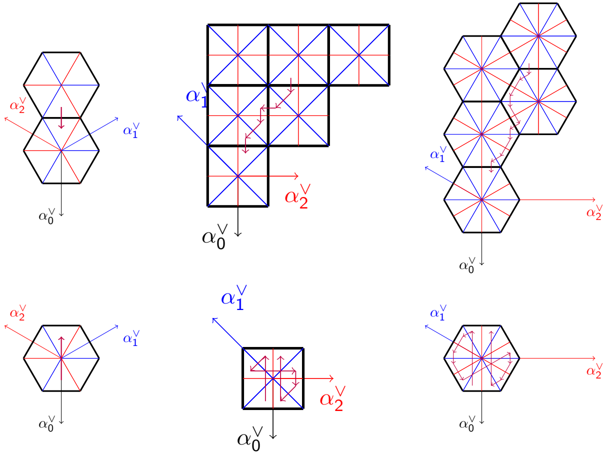

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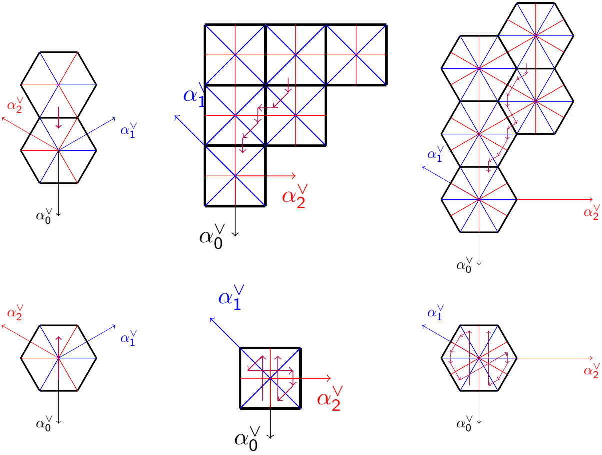

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Hecke group algebras and principal series representations

- $Y^{\lambda^\vee} \in \mathrm{H}(W)(q)$ (analog of translations in W)
- $\mathbb{C}[Y]$: commutative algebra, $\mathbb{C}[Y]^{\mathring{W}}$: center of $\mathrm{H}(W)(q)$
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  - Central specialization of $\mathbb{C}[Y]^W$ on t:
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    Quotient of $\mathcal{H}(q,t)$, generically trivial

Theorem (S.,T., 2008)

Take q non zero and non root of unity, and $t : Y^{\lambda^\vee} \mapsto q^{-\mathrm{ht}(\lambda^\vee)}$
Then, $\rho_t(\mathrm{H}(W)(q)) = \mathrm{H}\mathring{W}$
I.e. $\mathrm{H}\mathring{W}$ (non trivial!) quotient of $\mathcal{H}(q,t)$ and of $\mathrm{H}(W)(q)$

- Proof: diagonalization of the action of Y on $\mathbb{C}\mathring{W}$, using alcove walks and the intertwining operators τ_i
- What happens at roots of unity? (Nicolas Borie)

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Conclusion

- Hecke group algebras:
  - Many equivalent definitions
  - Nice structure and representation theory
  - (type A) Connections with NCSF, parking functions
  - Connections with 0-Hecke and affine Hecke algebras
- Where does this structure come from?
- Is it useful?