*-Combinat
Sharing algebraic combinatorics since 2000

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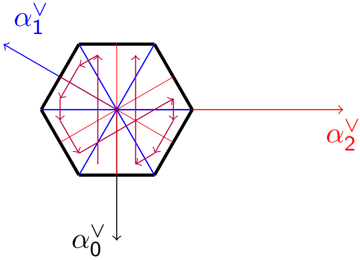
et al.

Sage Days 10, October 10-th of 2008



*-Combinat
An open-source framework
For computer exploration
In algebraic combinatorics

*-Combinat in a nutshell

Demo: combinatorics for all

Computer exploration in algebraic combinatorics: Cayley trees

Short history, switch to Sage

Status of the switch, advanced demo, future directions

It all started there

*-Combinat figures

- Community: Abbad, Bandlow, *Borie*, Bump, Carré, Chapotton, Denton, Descouens, Gomez-Diaz, *Hansen*, Hemmecke, *Hivert*, Jones, Laugerotte, Lecouvey, Lemeur, *Miller*, Molinero, Musiker, Novelli, Nzeutchap, Qiang, Rubey, Schilling, Shimozono, *Thiéry*, *Tevlin*, Walker, Zabrocki, *Zimmermann*
- End result: 50 publications
- 600 classes, 5000 methods, 130k lines of MuPAD
  60k lines of Sage
- 32k lines of tests, 600 pages of doc

Demo: combinatorics for all

Algebraic combinatorics

Motivation: relations algorithms ↔ algebraic structures

Combinatorial objects appear in algebraic computations

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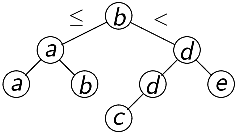


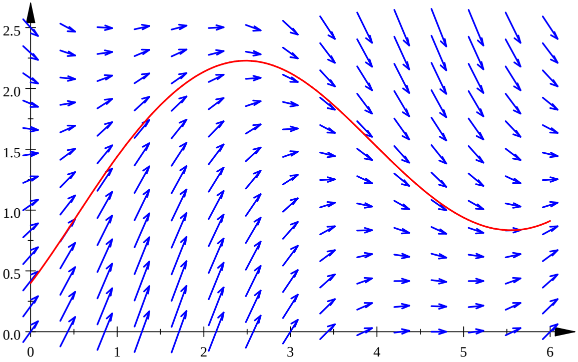
5			
2	4	8	
1	3	6	7

The (recursive) structure of the combinatorial objects encodes the algebraic structure.

Cayley: *On the theory of the analytical forms called trees*

Given a *vector field* $\vec{V}_x$, $x \in \mathbb{R}^d$
Find the *flow* integrating the vector field, *i.e.*, $x(t)$ such that:

$$x(0) = x_0 \qquad \text{and} \qquad x'(t) = \vec{V}_{x(t)}$$



The differential of a vector field

Let $\vec{V}$ and $\vec{U}_1, \ldots, \vec{U}_k$ be vector fields
The k-th differential $D^k\vec{V}$ of $\vec{V}$ is defined by

$$[D^k\vec{V}(\vec{U}_1, \ldots, \vec{U}_k)]_i := \sum_{j_1 \ldots j_k = 1}^{d} \frac{\partial^k [\vec{V}]_i}{\partial x_{j_1} \ldots \partial x_{j_k}} [\vec{U}_1]_{j_1} \ldots [\vec{U}_k]_{j_k} \,,$$

where $[\vec{W}]_i$ denotes the i-th coordinate of the vector field $\vec{W}$

$D^k\vec{V}$ maps k vector fields to one vector field

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The derivatives of $x(t)$

$$\frac{dx}{dt}(t) = \vec{V}_{x(t)} = (\vec{V} \circ x)(t)$$

Using the chain rule (derivative of composed functions):

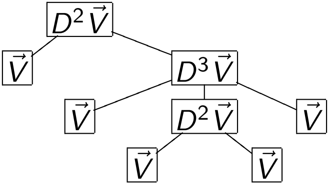
$$\frac{d^2x}{dt^2} = D\vec{V}_x\left(\frac{dx}{dt}\right) = D\vec{V}_x(\vec{V}_x)$$

Third and fourth derivative with implicit $x(t)$:

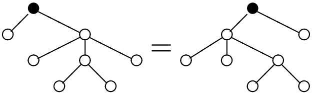
$$\begin{aligned}\frac{d^3x}{dt^3} =& D^2\vec{V}(\vec{V},\vec{V}) + D\vec{V}(D\vec{V}(\vec{V}))\\ \frac{d^4x}{dt^4} =& D^3\vec{V}(\vec{V},\vec{V},\vec{V}) + 3\,D^2\vec{V}(\vec{V},D\vec{V}(\vec{V})) +\\ & D\vec{V}(D^2\vec{V}(\vec{V},\vec{V})) + D\vec{V}(D\vec{V}(D\vec{V}(\vec{V})))\end{aligned}$$

A better notation: expression trees

$$D^2\vec{V}(\vec{V}, D^3\vec{V}(\vec{V}, D^2\vec{V}(\vec{V}, \vec{V}), \vec{V})) =$$



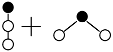
Clairaut's theorem $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$: rooted topological trees

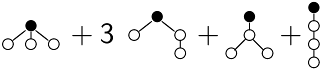


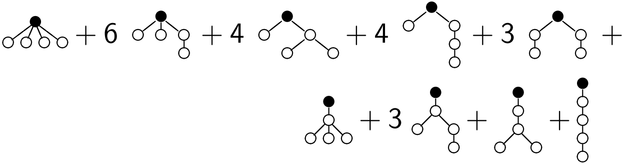
The derivatives of $x(t)$ (continued)

$$\frac{dx}{dt} =$$ 

$$\frac{d^2x}{dt^2} =$$ 

$$\frac{d^3x}{dt^3} =$$ 

$$\frac{d^4x}{dt^4} =$$ 

$$\frac{d^5x}{dt^5} =$$ 

$\dots$

Closed formula

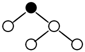
Theorem

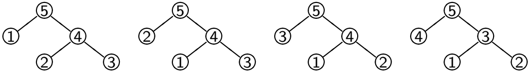
The n-th derivative of $x(t)$ is given by

$$\frac{d^n x}{dt^n} = \sum_{T:\ \text{tree of size } n} c_T\, T$$

where c_T is the number of standard decreasing labelings of T.

Example:

c  $= 4$:

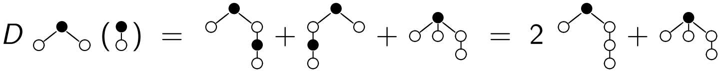


Some algebraic structure

Pre-Lie operation: $(\vec{U}, \vec{V}) \mapsto \vec{U} * \vec{V} := D\vec{U}(\vec{V})$

$$DT_1(T_2) = \sum_{n:\text{node of } T_1} \text{grafting of the root of } T_2 \text{ on } n$$

For example:



Pre-Lie Identity:

$$(x * y) * z - x * (y * z) = (x * z) * y - x * (z * y)$$

Requirements for a typical computation

- A bit of standard combinatorics
- A bit of standard linear algebra
- A bit of standard group theory
- A bit of standard computer algebra
- A bit of standard ...
- And your own little touch

There is nothing like a complete algebraic combinatorics package

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Importance of good and consistent large-scale design

*-Combinat is all about

- Sharing

  Remember: this goes both ways!

- Building a community

- Integration: a well organized repository body of code (?)

- Using a bit of computer science
  To organize the code and the community
  To put a bit more math in the machine

- Doing research!

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2000: state of the art

Algebraic combinatorics packages

- guess, combstruct, gfun, CS (Projet Algo, INRIA)
- SF (Stembridge)
- ACE, μ-EC (Marne-la-Vallée)
- Symmetrica (Bayreuth)
- Rate, ...

Platforms

- Maple / Maxima
- GAP
- Magma
- Axiom / Aldor
- MuPAD

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2007: The MuPAD-Combinat team in action

2008: How to scale further?

- Design thingies, speed, ...
- Architecture, modeling, ...
- Identifying highest return value algorithms

Our community was not broad enough

- Sharing did not yet pay off (for me)!
- Too much management work
- Too much engineering underground work
- Too much off topic work

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The platform was not broad enough

- MuPAD is not a widely used language and system
- MuPAD is not open-source

Dealing with it:

- 1999: Negotiation with the MuPAD team
- 2002: Open source computer algebra workshop
- 2005: Sage on radar
- 2006-2007: Axiom meetings → Aldor-Combinat
- 2007: Mike on radar
- February: Sage Days 7
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Concentrate on the foundations

- Counting functions / generating series $\surd$?
  lazy Karatsuba product, plethysm, implicit equation, ...
- Generators / iterators, continuations (yield) $\surd$
- Data structures for combinatorial objects
  partitions$\surd$, graphs$\surd$, trees, tableaux$\surd$, permutations$\surd$, ...)
- Lexicographic enumeration of list of integers $\surd$?
- Integral points of a polyhedron $\surd$?
- Decomposable classes / species (Hansen) $\surd$
- Objects mod a group action (Miller) $\surd$?

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- Posets and linear extensions (Saliola, Hivert, Nzeutchap) √?
- Words (Saliola)
- Root systems, crystals, Weyl groups√
  (Schilling, Bandlow, Bump, Walker, Hansen, T)
- (Weighted) Automatons (Saliola, Laugerotte)
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*-Combinat building steps (algebra)

*-Combinat building steps (algebra I)

Trivial code that solves trivial problems is precious

- `FreeModule(Combinatorial Class, Coefficient Ring)`$\surd$
- Categories: (Hopf)AlgebraWithBasis and friends
  Unification with polynomials, Weyl & Ore algebras, ...
- Seamless linear algebra (Linbox)
  Hom(..., ...)
- Spaces with several representations, ...
- Functors: tensor products, tensor, exterior, and symmetric algebra, submodules, quotients, ...

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- Spaces with several representations, ...
- Functors: tensor products, tensor, exterior, and symmetric algebra, submodules, quotients, ...

∗-Combinat building steps (algebra I)

Trivial code that solves trivial problems is precious

- `FreeModule(Combinatorial Class, Coefficient Ring)` $\surd$
- Categories: (Hopf)AlgebraWithBasis and friends
  Unification with polynomials, Weyl & Ore algebras, ...
- Seamless linear algebra (Linbox)
  Hom(..., ...)
- Spaces with several representations, ...
- Functors: tensor products, tensor, exterior, and symmetric algebra, submodules, quotients, ...

*-Combinat building steps (algebra II)

- Symmetric functions (Hansen) √?
- Non commutative and quasi-symmetric functions (Tevlin, Luoto)
- Database of Hopf algebras
- Operads (Chapoton, Saliola)
- Permutation groups (GAP) √
- Quivers (Lemeur?)
- Generic Gröbner/Involutive elimination tools
- Representation Theory (Hivert)
- Crystals, Weyl groups, ... √?

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*-Combinat building steps (coercion)

- We need automatic coercion (implicit conversions)
- At interactive-level *and* inside code
- Possibly with > 1000 domains simultaneously

A good test case for the new Sage coercion model!

Want to join?

mupad-combinat.sf.net

wiki.sagemath.org/combinat