Sorting monoids on Coxeter groups

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arXiv:0711.1561 [math.RT] (FPSAC'06)
arXiv:0804.3781 [math.RT] (FPSAC'08)
arXiv:0912.2212 [math.CO] (FPSAC'10)
+ research in progress

Bubble (anti) sort algorithm

1234

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1243

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1423

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4123

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Underlying combinatorics: right permutahedron

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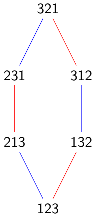

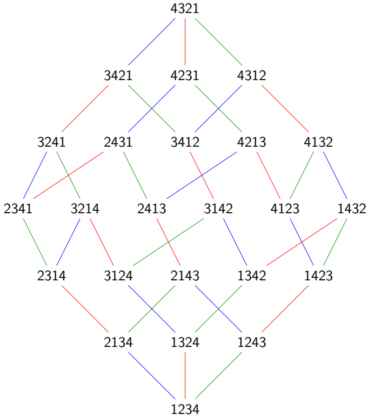

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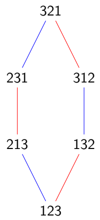
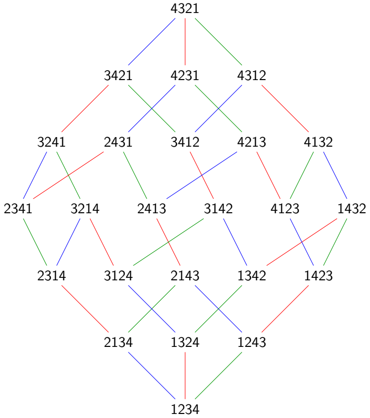
Underlying combinatorics: right permutahedron





Elementary transpositions: $s_1, s_2, s_3, \ldots$

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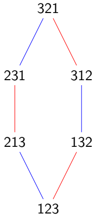
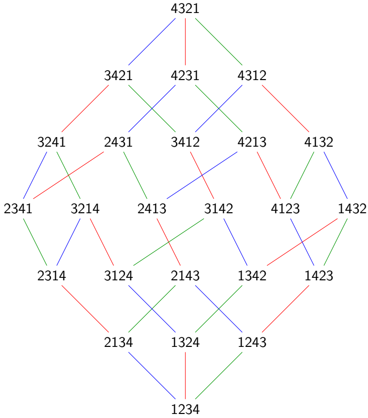




Elementary transpositions: $s_1, s_2, s_3, \ldots$

Relations: $s_i^2 = 1, (s_1 s_2)^3 = 1, (s_2 s_3)^3 = 1, (s_1 s_3)^2 = 1$

Coxeter groups

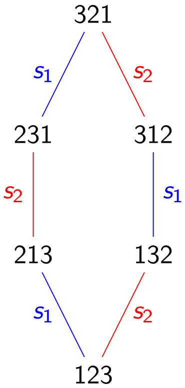
Definition (Coxeter group W)

Generators : $s_1, s_2, \ldots$ (simple reflections)

Relations: $s_i^2 = 1$ and $\underbrace{s_i s_j \cdots}_{m_{i,j}} = \underbrace{s_j s_i \cdots}_{m_{i,j}}$, for $i \neq j$

Reduced words

0-Hecke monoid



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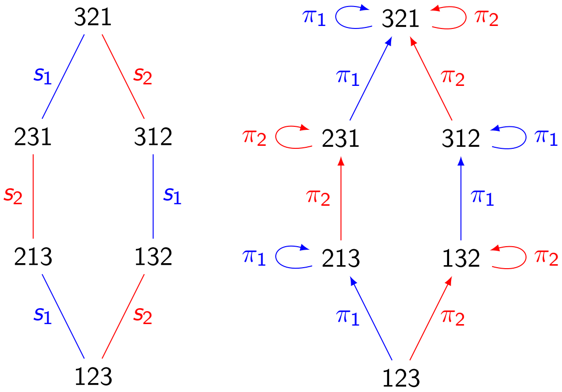

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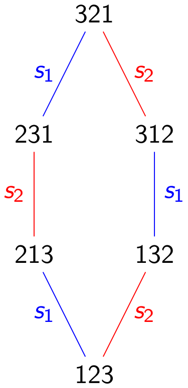

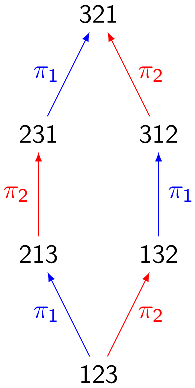

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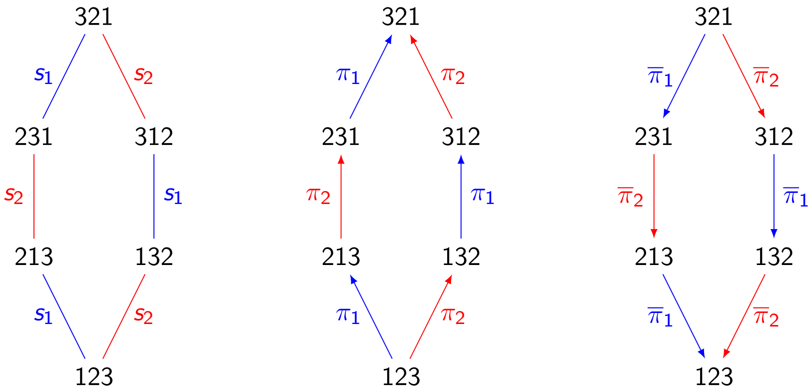

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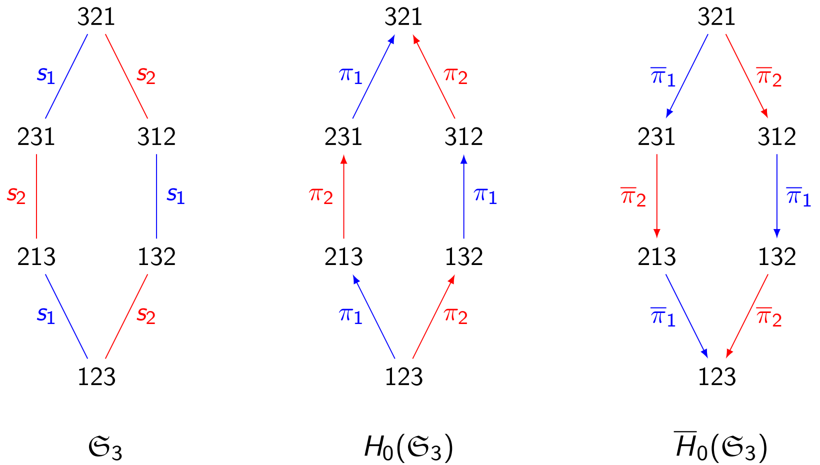

0-Hecke monoid

Definition (0-Hecke monoid $H_0(W)$ of a Coxeter group W)

Generators : $\langle \pi_1, \pi_2, \dots \rangle$ (simple reflections)
Relations: $\pi_i^2 = \pi_i$ and braid relations

Theorem

$|H_0(W)| = |W|$
+ lots of nice properties

Motivation: simple combinatorial model (bubble sort)
appears in Iwahori-Hecke algebras, Schur symmetric functions, Schubert, Macdonald, Kazhdan-Lusztig polynomials, (affine) Stanley symmetric functions, mathematical physics, Schur-Weyl duality for quantum groups, representations of $GL(\mathbb{F}_q)$, ...

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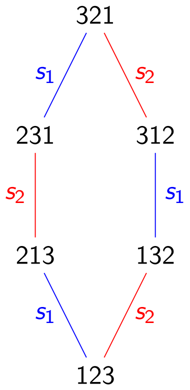
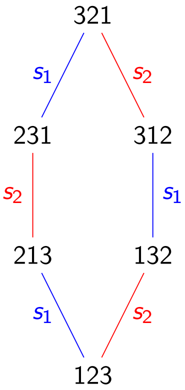


Right order

Classical orders on Coxeter groups



Right order

Prefix

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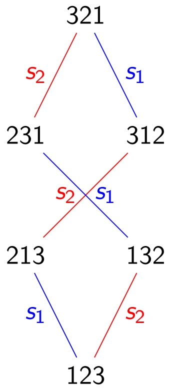

Left order

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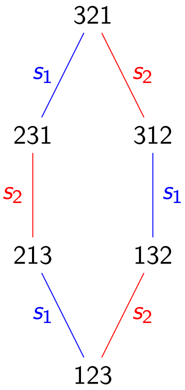

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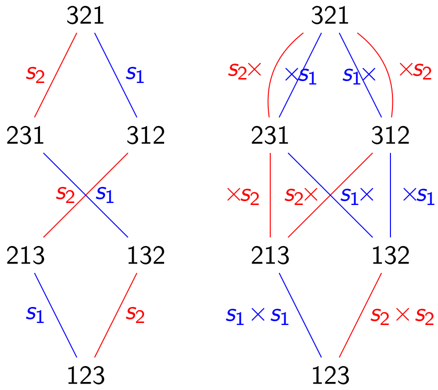
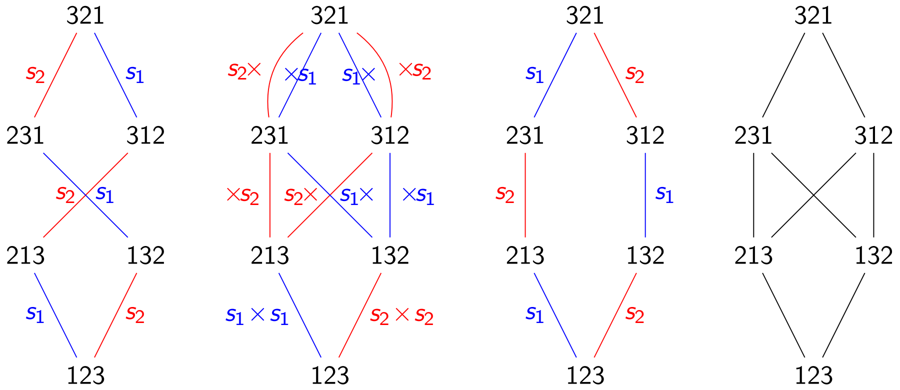


Left order	Left-Right order	Right order
Suffix	Factor	Prefix

Classical orders on Coxeter groups



Left order	Left-Right order	Right order	Bruhat order
Suffix	Factor	Prefix	Subword

Blocks of permutations

Definition (Block of a permutation w)

- Type A: sub-permutation matrix
- Type free: J, K such that $W_J w = w W_K$

- Example: $w := 36475812$

- Simple permutation: cf. [Albert, Atkinson 05] + dim 2 posets
- $\{\texttt{blocks of } w\}$: sub-lattice of the Boolean lattice

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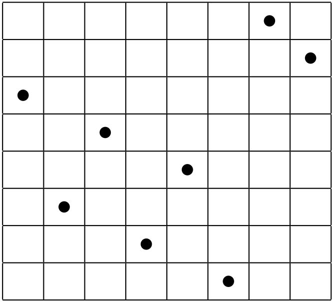

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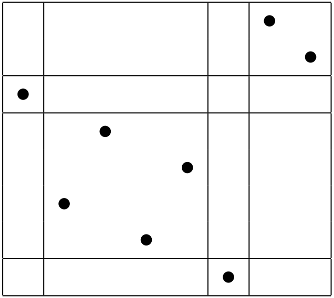

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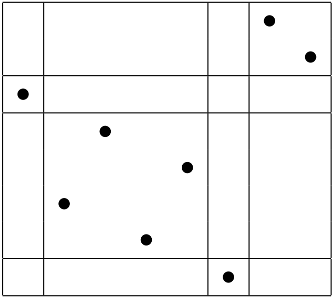

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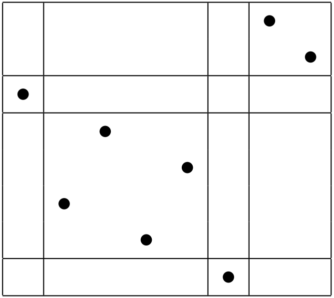

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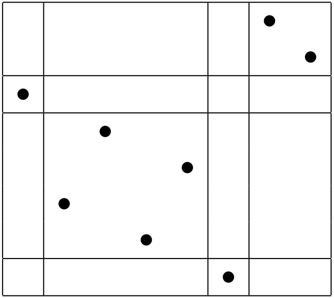

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The cutting poset

Definition (HST09: Cutting poset $(W, \sqsubseteq)$)

$u \sqsubseteq w$ if $u = w^J$ with J block

Theorem

- *Intervals are lattices*
- *Möbius function: inclusion-exclusion along minimal blocks*

- Meet-semi lattice?

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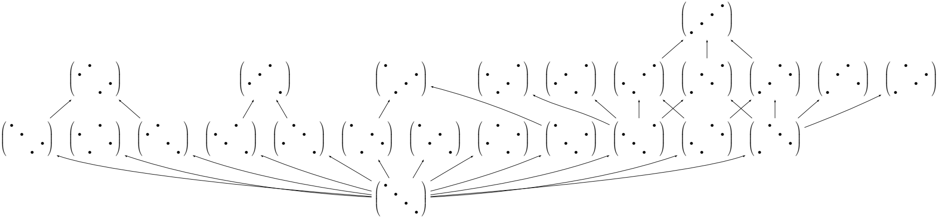

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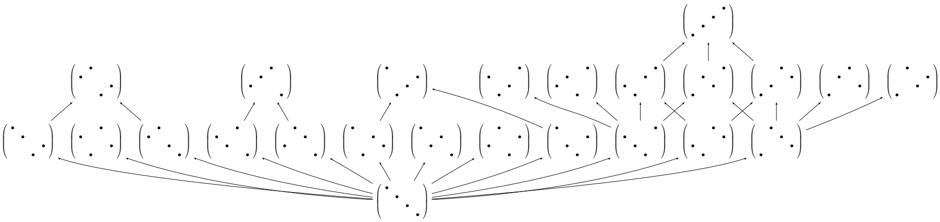

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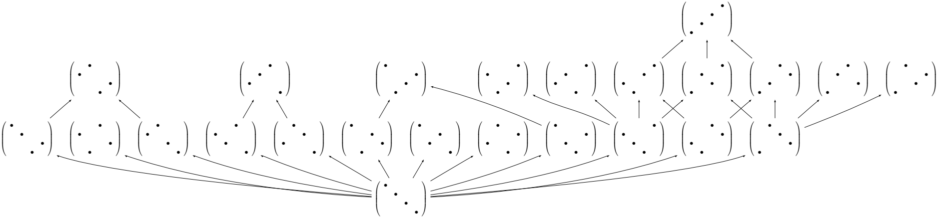

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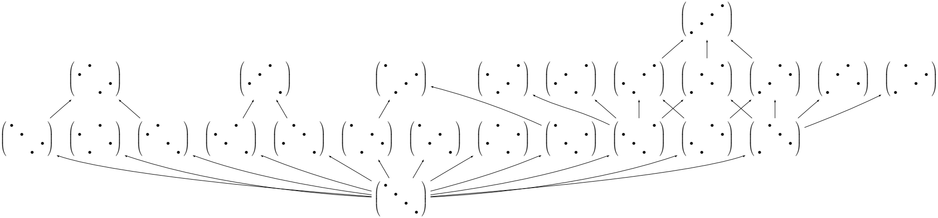

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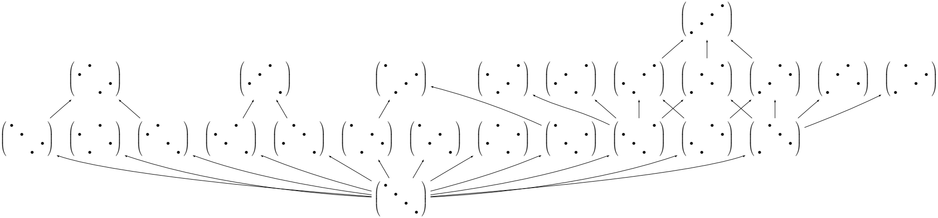

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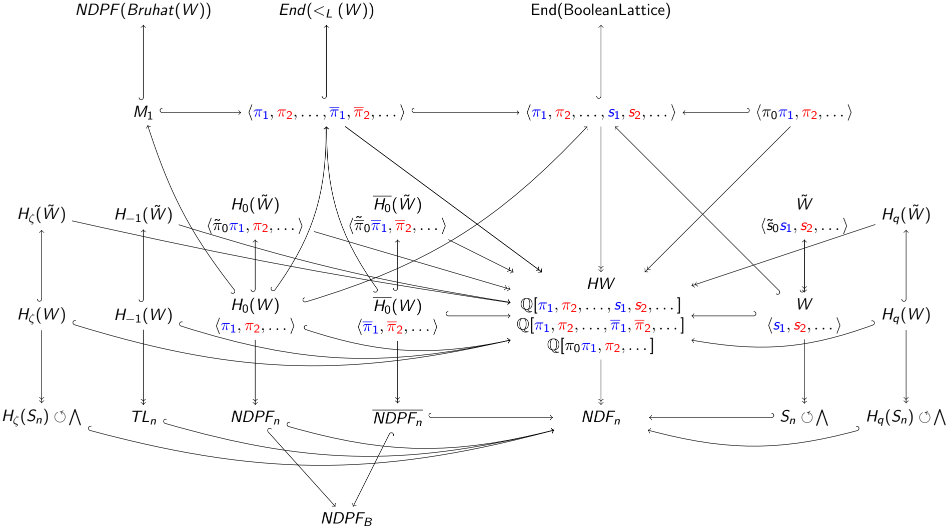
The Big Picture



The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$

$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

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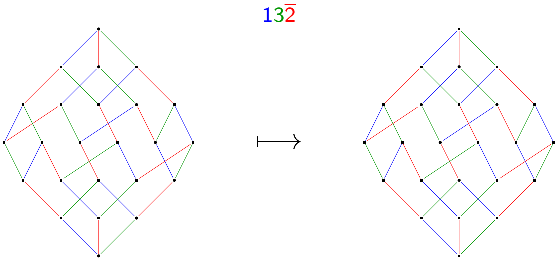
Key combinatorial lemma



Lemma

For $f \in M(W)$ and $w \in W$: $\quad (s_i w).f = w.f \quad$ *or* $\quad s_i\,(w.f)$

Proof.

Exchange property / associativity □

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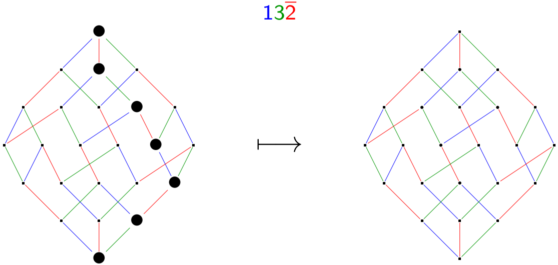

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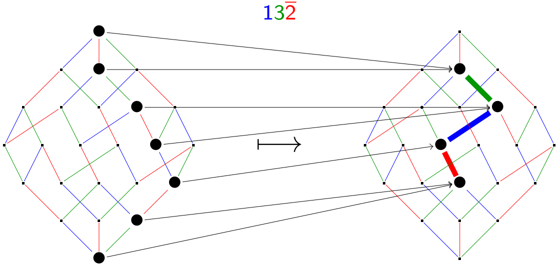

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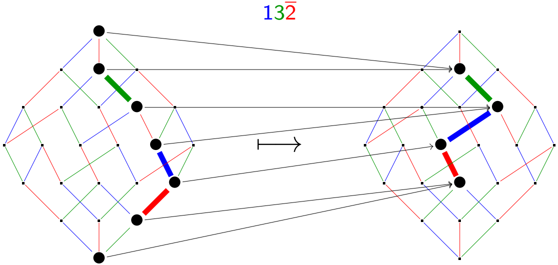

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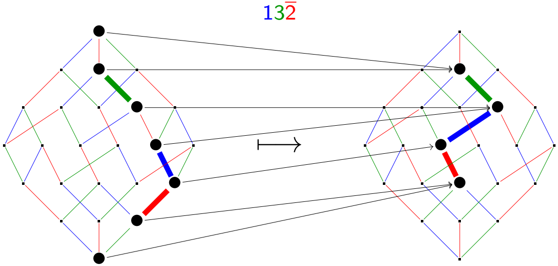

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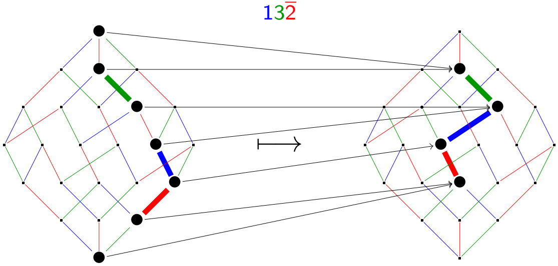

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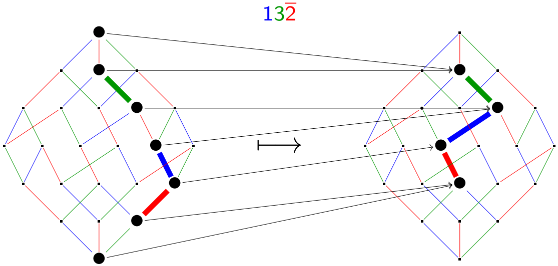

Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- $M(W)$ *is aperiodic*
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

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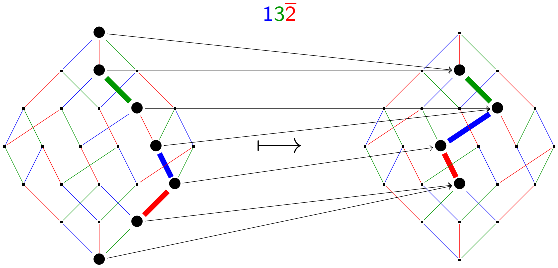

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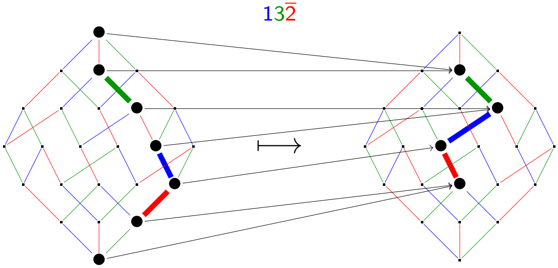

Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
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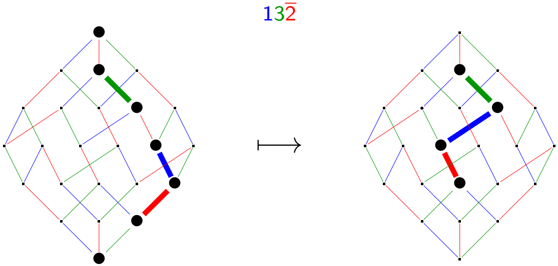

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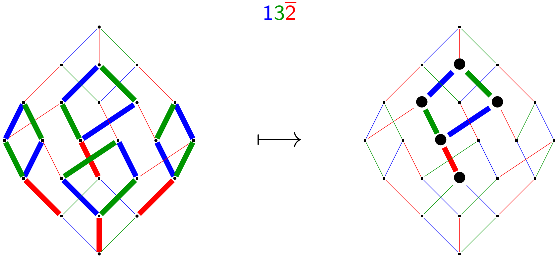

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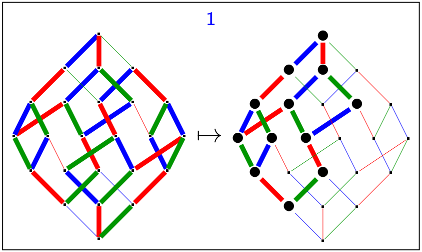
Some elements of the monoid



Lemma

The image set of an idempotent is an interval in left order

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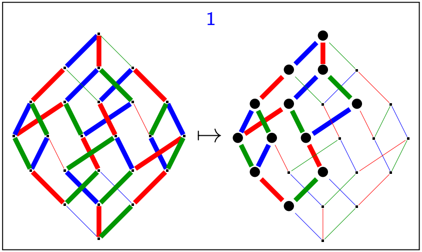

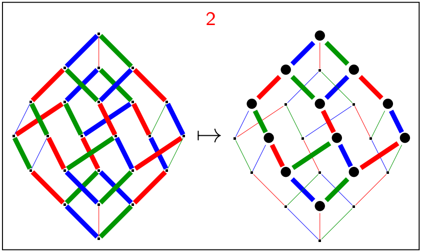

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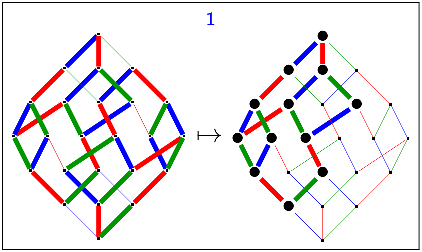

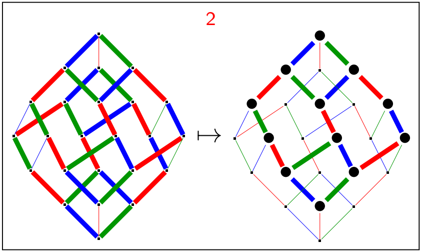

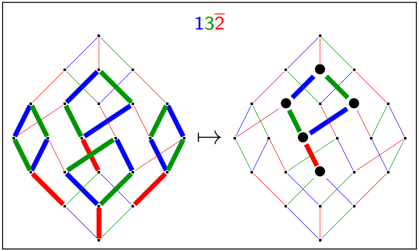

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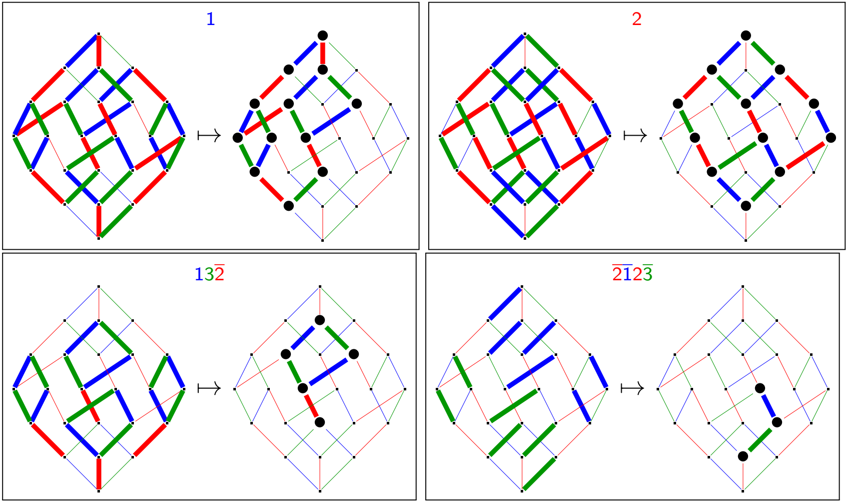

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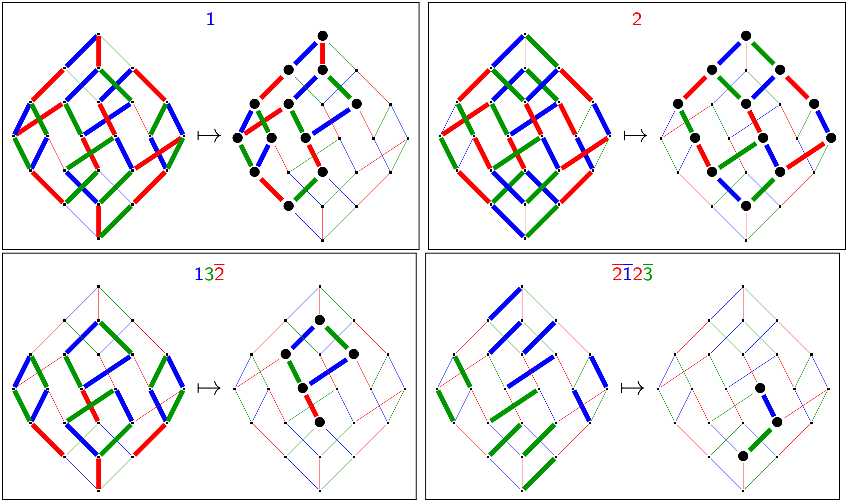

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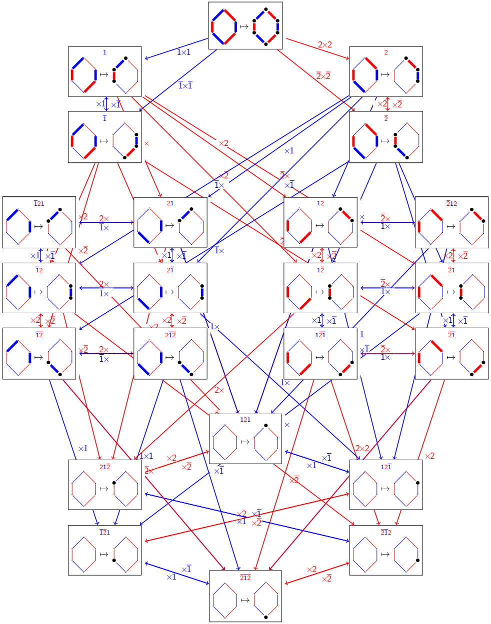
Green relations



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Theorem

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
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- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

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Representation theory of $M(W)$

Corollary

$M(W)$ admits $|W|$ simple modules / indecomposable projective modules

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Dimension of simple and indecomposable projective modules?

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The “Borel” submonoid M_1

Definition

Submonoid $M_1 := \{f \in M, f(1) = 1\}$

Properties

- *Weakly increasing and contracting on Bruhat* $\Longrightarrow$ $\mathcal{J}$*-trivial*
- *Idempotents:* $(e_w)_{w \in W}$
- *Generated by* e_w *for* w *grassmanian, e.g. atom for* $(W, \vee_L)$
- $|W|$ *simple modules of dimension* 1
- *Semi-simple quotient: monoid algebra of* $(W, \vee_L)$
- *Conjugacy order among idempotents:* $<_L$
- $\dim P_w = |\{f \in M_1,\ f(w) \leq_L w\}| =?$

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Inducing these results to M*?*

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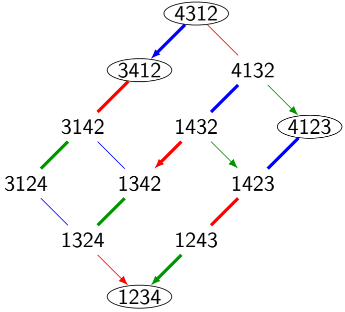
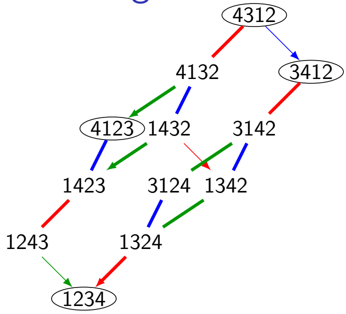
$\mathcal{R}$-classes and translation algebras





Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \dots, \overline{\pi}_1, \overline{\pi}_2, \dots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1,2\}, \{3\}, \{1,2,3\}$ $\implies$ Submodules P_J
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- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
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- Generating series calculation?

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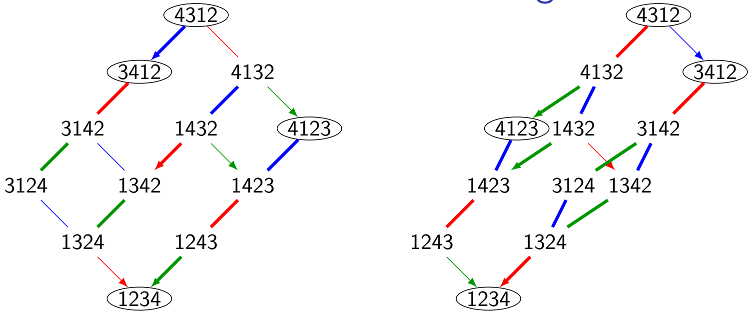

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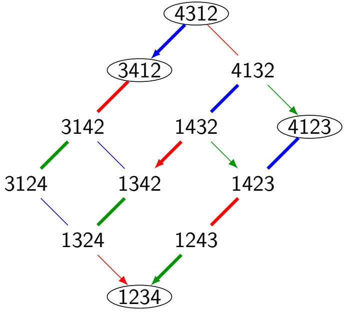

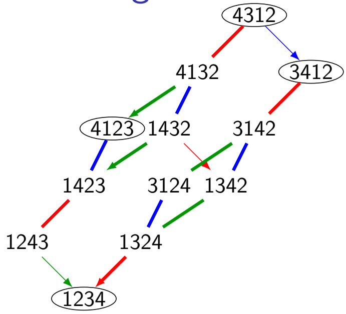

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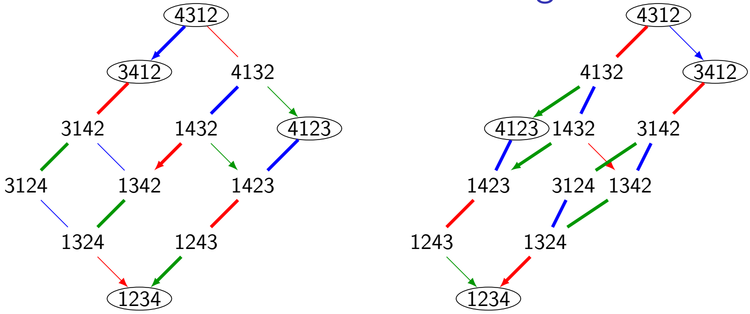

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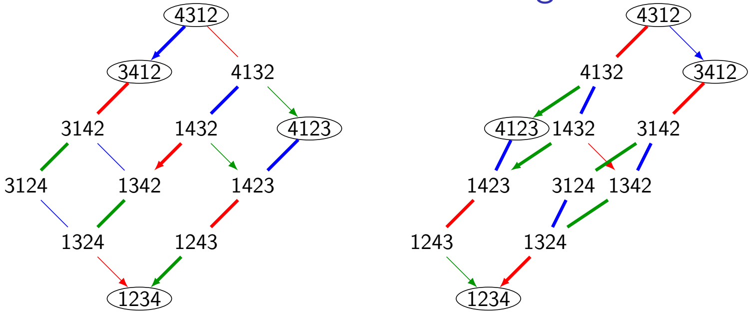

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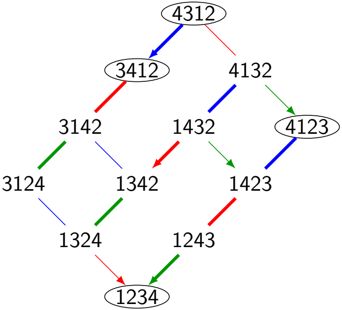

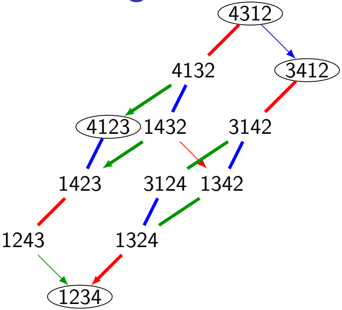

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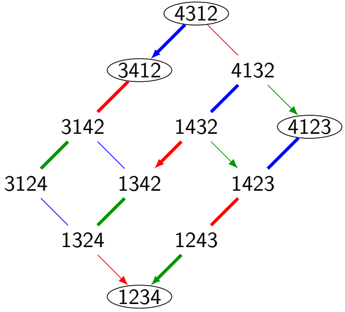

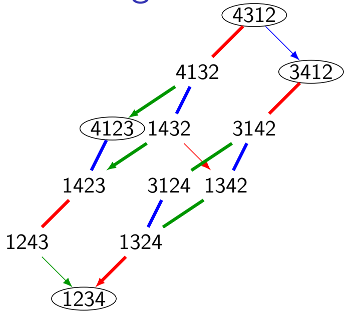

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Work in progress

- $\mathcal{L}$-classes? Projective modules? Cartan Matrix?
- Generalization to $\mathcal{R}$-trivial and aperiodic monoids (collaboration with Denton and Berg, Bergeron, Saliola)
- Fast implementation in Sage (interface with `Semigroupe`, ...)

Sage-Combinat meeting tonight

Sage's mission:

"To create a viable high-quality and open-source alternative to Maple™, Mathematica™, Magma™, and MATLAB™"

...

"and to foster a friendly community of users and developers"

Tonight, Thorton Hall, Room 326

- 7pm-8pm: Introduction to `Sage` and `Sage-Combinat`
- 8pm-10pm: Help on installation & getting started
  Bring your laptop!
- Design discussions