Combinatoire et représentations des monoïdes de tris

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arXiv:0711.1561 [math.RT]
arXiv:0804.3781 [math.RT]
arXiv:0912.2212 [math.CO]
arXiv:1010.3455 [math.RT]
arXiv:1012.1361 [math.CO]
+ research in progress

Bubble (anti) sort algorithm

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4123

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Underlying combinatorics: right permutahedron

Bubble (anti) sort algorithm

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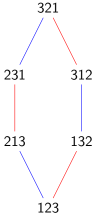
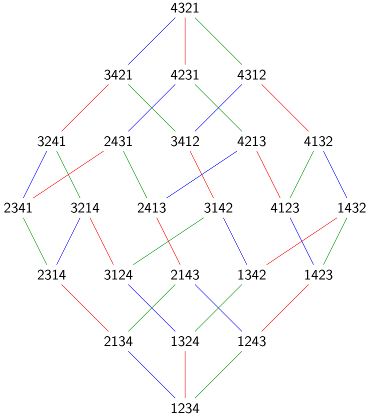
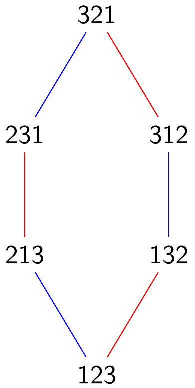
Underlying combinatorics: right permutahedron





The permutohedron, as an automaton



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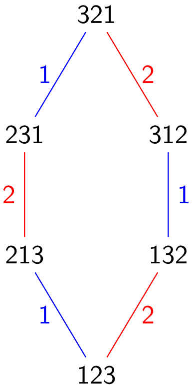

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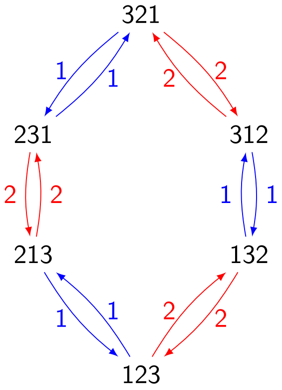

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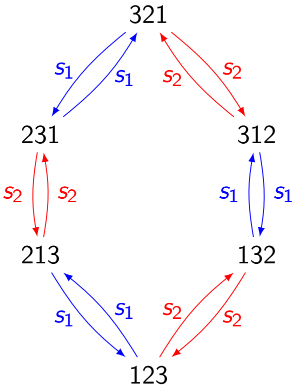

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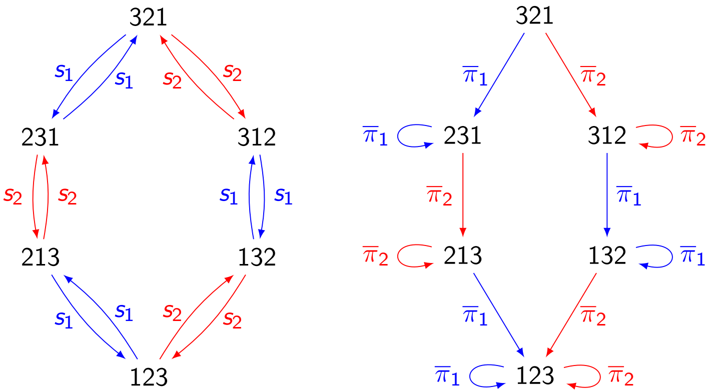

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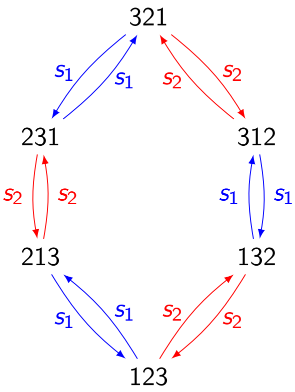

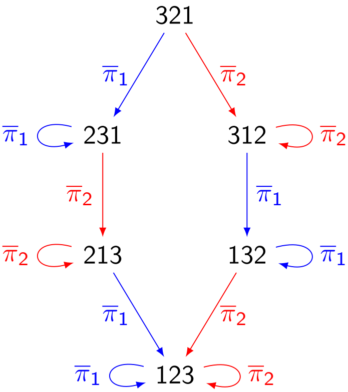

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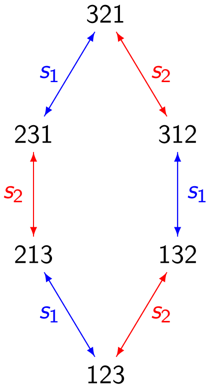

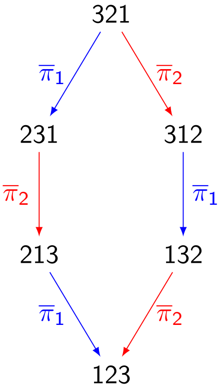

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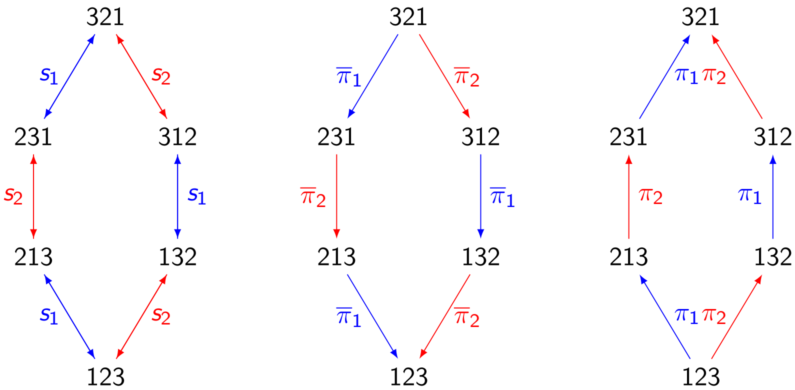


The transition monoid of an automaton

An automaton

A set of states: Q
An alphabet: $A = \{a_1, \ldots, a_n\}$
A set of transitions: $q \xrightarrow{a_i} q'$

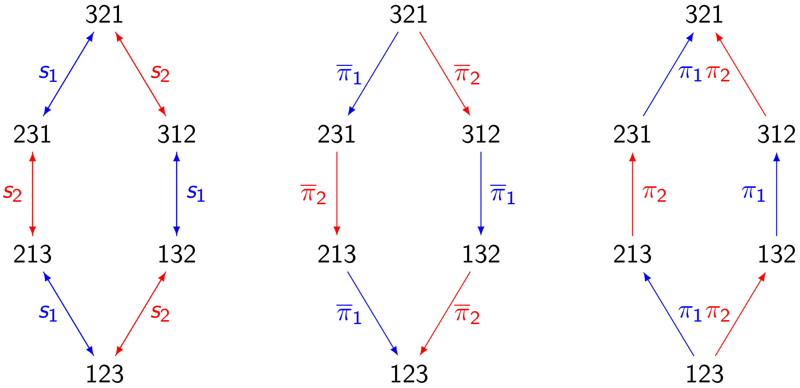
The free monoid and its action on the automaton

The free monoid A^*: words w over A, concatenation product
ρ maps a word w to a function $f_w : Q \mapsto Q$

The transition monoid

The monoid $\rho(A^*)$ generated by $f_{a_1}, \ldots, f_{a_n}$

Monoids associated to the permutohedron



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$$s_1 s_2 s_1 = s_2 s_1 s_2$$

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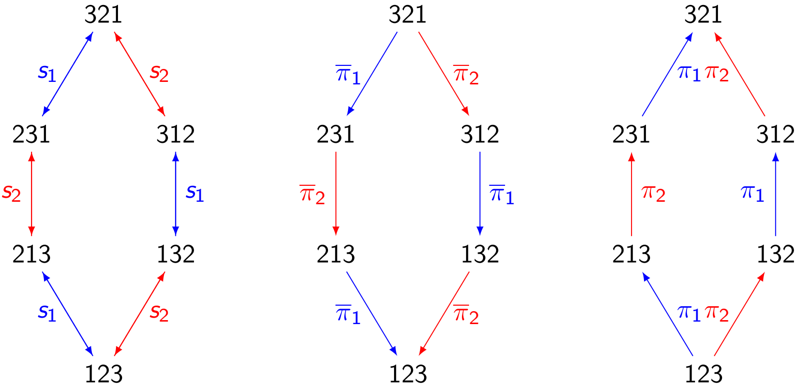

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Symmetric group $\mathfrak{S}_3$

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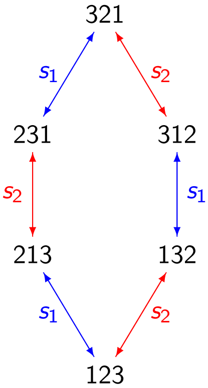

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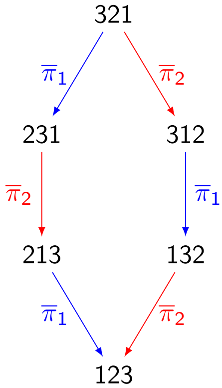

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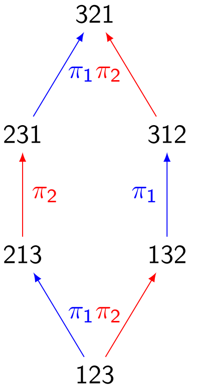

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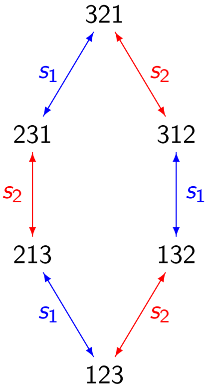

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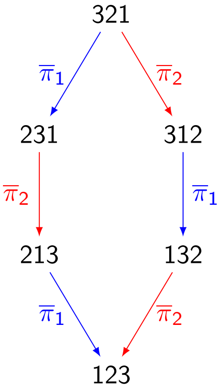

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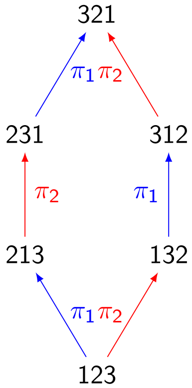
0-Hecke monoide $\overline{H}_0(\mathfrak{S}_3)$



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$H_0(\mathfrak{S}_3)$

Coxeter groups

Definition (Coxeter group W)

Generators : $s_1, s_2, \dots$ (simple reflections)

Relations: $s_i^2 = 1$ and $\underbrace{s_i s_j \cdots}_{m_{i,j}} = \underbrace{s_j s_i \cdots}_{m_{i,j}}$, for $i \neq j$

Reduced words

0-Hecke monoid

Definition (0-Hecke monoid $H_0(W)$ of a Coxeter group W)

Generators : $\langle \pi_1, \pi_2, \dots \rangle$ (simple reflections)
Relations: $\pi_i^2 = \pi_i$ and braid relations

Theorem

$|H_0(W)| = |W|$
+ lots of nice properties

Motivation: simple combinatorial model (bubble sort)
appears in Iwahori-Hecke algebras, Schur symmetric functions, Schubert, Macdonald, Kazhdan-Lusztig polynomials, (affine) Stanley symmetric functions, mathematical physics, Schur-Weyl duality for quantum groups, representations of $GL(\mathbb{F}_q)$, probabilities, ...

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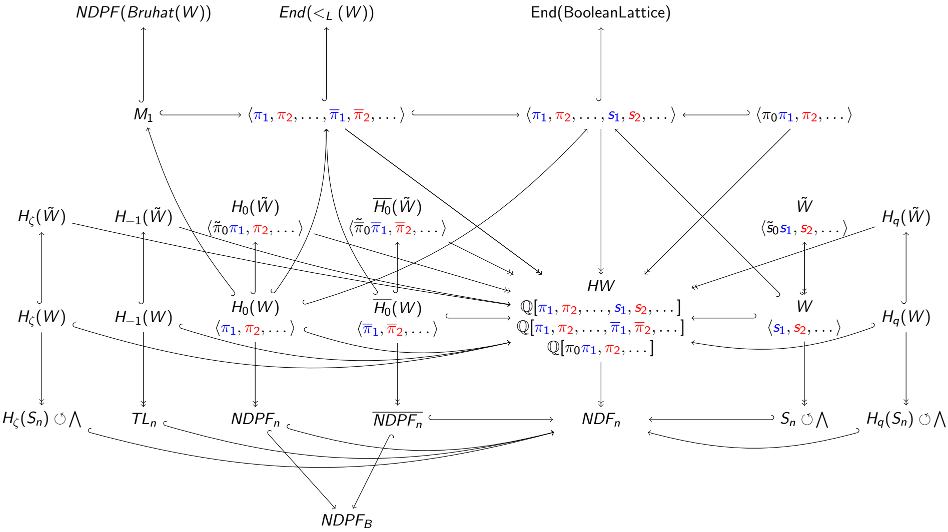
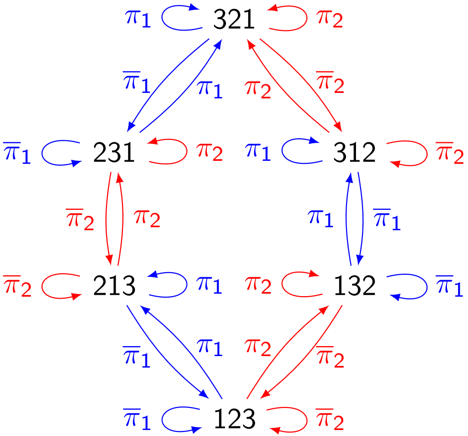
The Big Picture

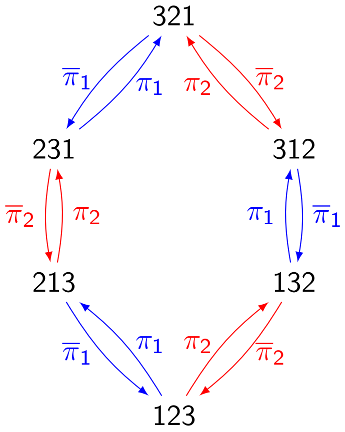


A strange cocktail: the biHecke monoid



What's the transition monoid?

A strange cocktail: the biHecke monoid



What's the transition monoid?

The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$

$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (Hivert, Schilling, T. '08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

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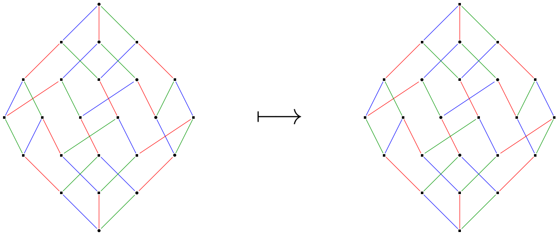
Key combinatorial lemma

$13\overline{2}$



Lemma

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Proof.

Exchange property / associativity □

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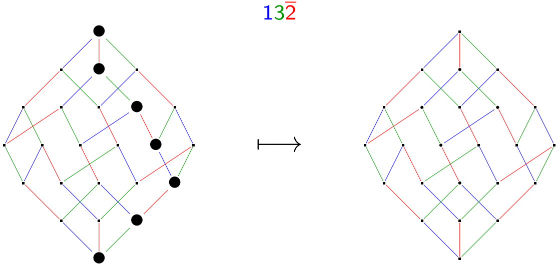

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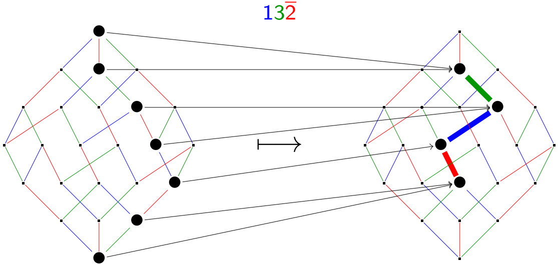

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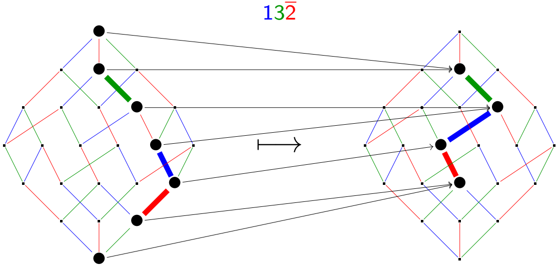

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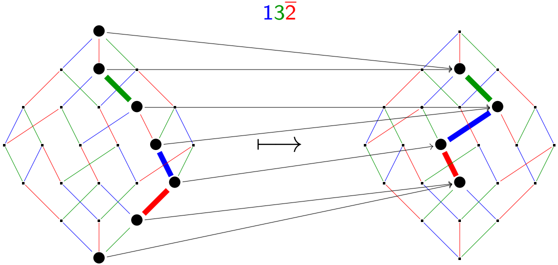

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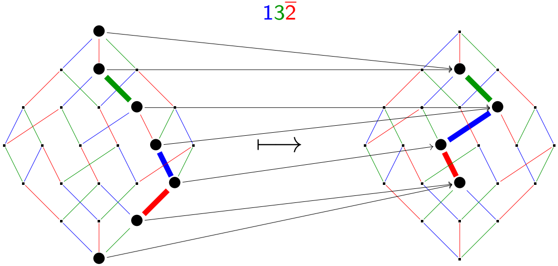

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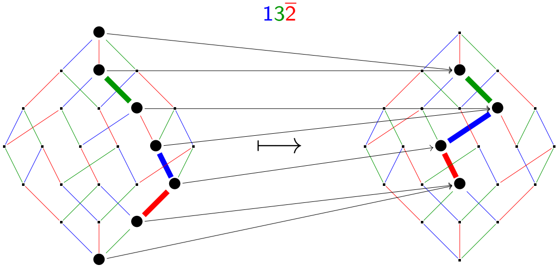

Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- $M(W)$ *is aperiodic*
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

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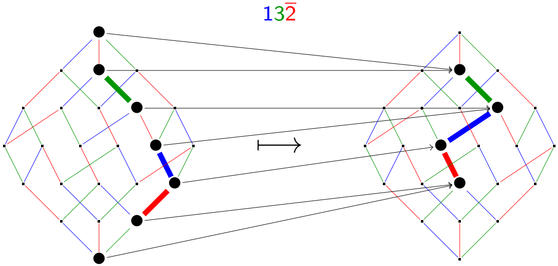

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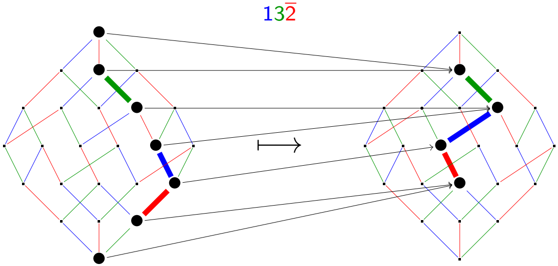

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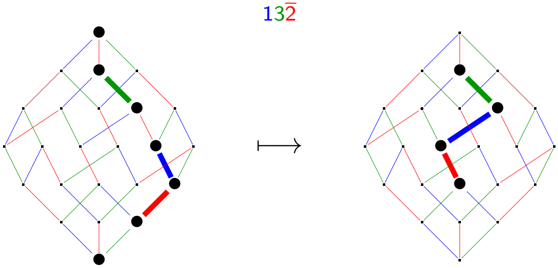

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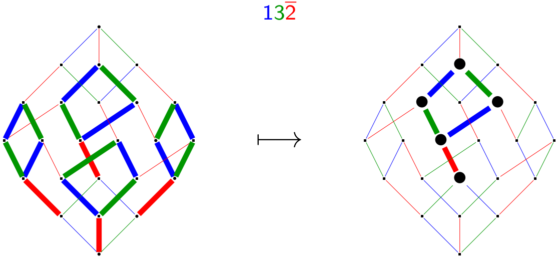

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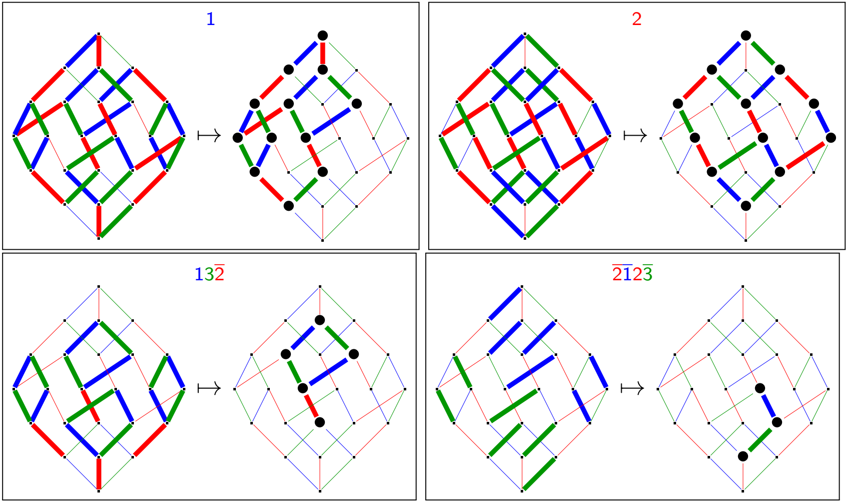
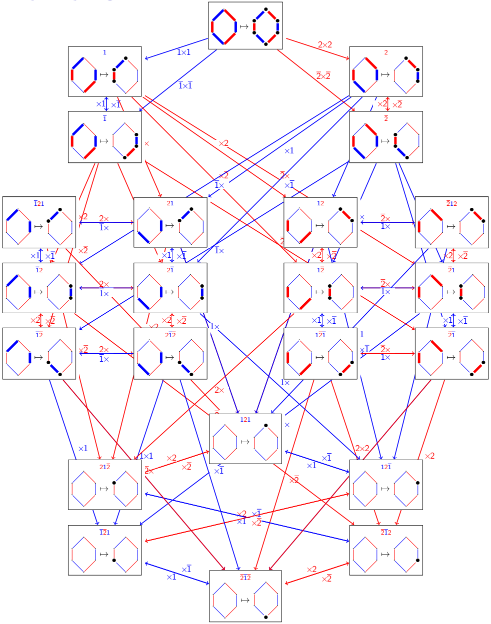
Some elements of the monoid



Lemma

The image set of an idempotent is an interval in left order

Cayley graph and Green relations



Green relations

Theorem (HST'08-10)

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
- *$\mathcal{L}$-order on regular $\mathcal{L}$-classes: $\approx$ left order on W*

Problems

- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

Representation theory

Definition

A: algebra / group / monoid

Linear representation: vector space V with a morphism

$$\rho \ : \ A \longmapsto \mathrm{End}(V)$$

(Left) Module: Bilinear operation $a.v$ (for $a \in A$, $v \in V$) such that

$$a.(b.v) = (ab).v$$

Idea:

- Better understanding of A via multiple views on it
- Reduce the study of A to linear algebra

Representation theory (building blocks)

Definition

Submodule $W \subset V$ is a stable subspace (if $x \in W$ then $a.x \in W$).

Simple (irreducible) module: no nontrivial submodule.

The smallest possible modules.
Indecomposable module: V cannot be written as $V = V_1 \oplus V_2$
Projective module: $V \oplus \cdots = \mathbb{C}[M] \oplus \cdots \oplus \mathbb{C}[M]$

Representation theory

Theorem (See e.g. Curtis-Reiner)

Simple modules $\leftrightarrow$ indecomposable projective modules
Dimension formula, ...

It's computational!

$\Rightarrow$ computer exploration

- Algebras: linear algebra $\gg O(n^3)$
- Groups: characters $o(n)$
- What about monoids?

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Theorem (See e.g. Curtis-Reiner)

Simple modules $\leftrightarrow$ *indecomposable projective modules*
Dimension formula, . . .

It's computational!

$\Rightarrow$ computer exploration

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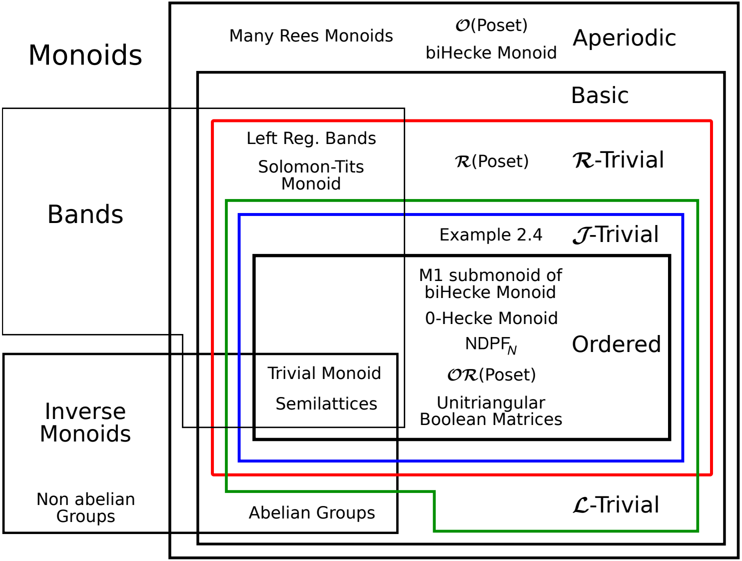

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Representation theory of $M(W)$

Theorem (HST'08)

$M(W)$ admits $|W|$ simple modules / indecomposable projective modules

Problem

Dimension of simple and indecomposable projective modules?

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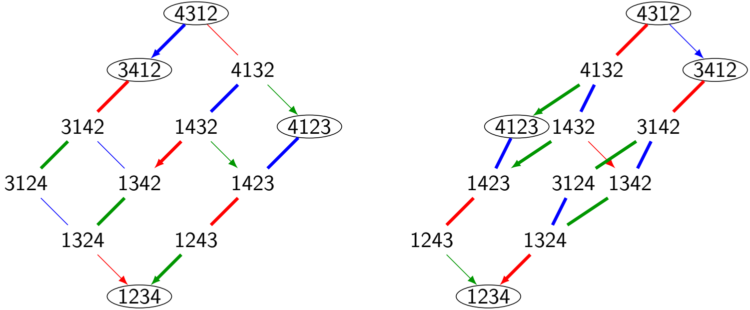
$\mathcal{R}$-class modules



- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\} \implies$ Submodules P_J
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- Dimension: inclusion-exclusion along the **cutting poset**
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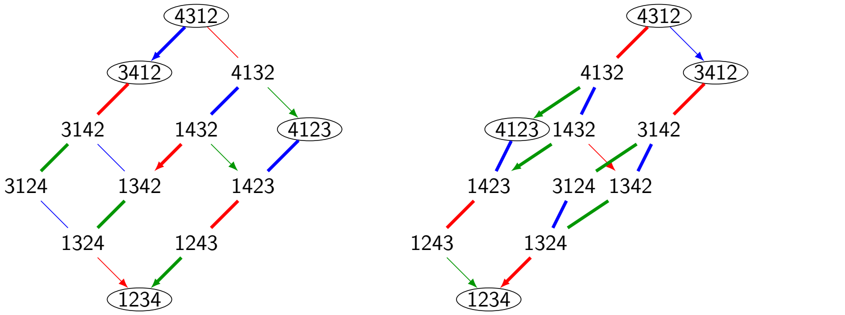

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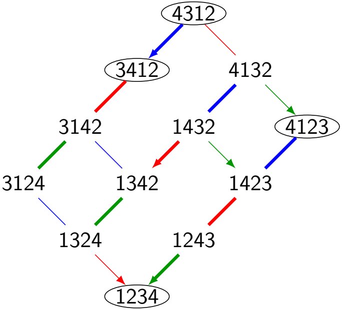

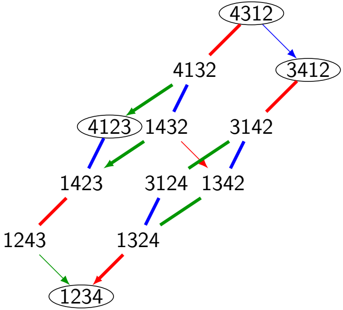

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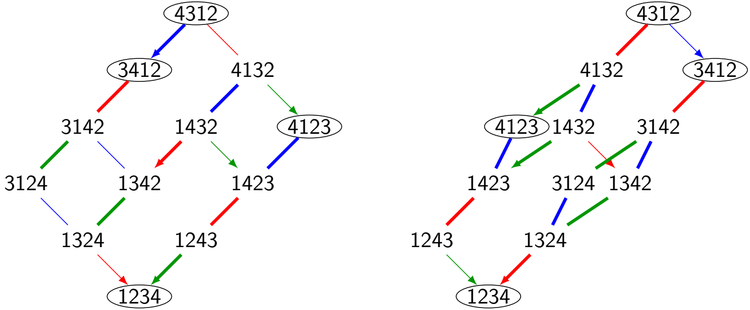

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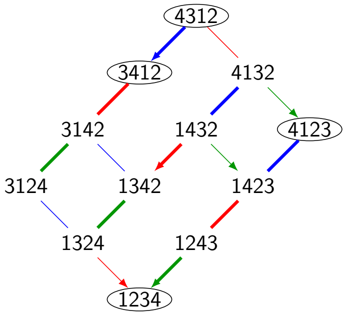

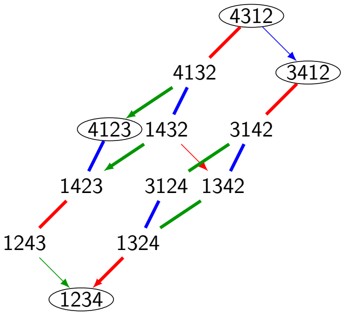

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$u \sqsubseteq w$ if $u = w^J$ with J block

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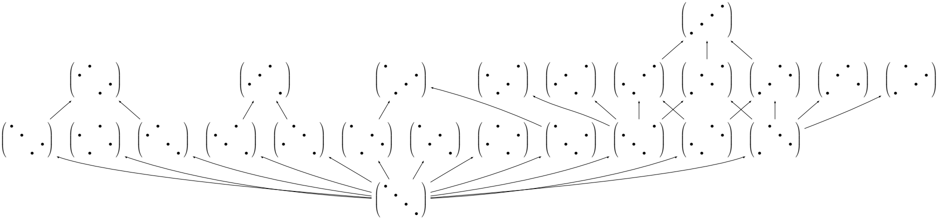

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- It is sufficient to look at left and right class modules
- Fast (mostly) combinatorial algorithms based on embeddings of automata

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Computer exploration

- Use algebra to raises questions
- Use combinatorics to make them concrete
- Use computer exploration to suggest answers and produce combinatorial data
- Analyse the data
- Find combinatorial regularities and proofs
- Derive conceptual interpretations
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The previous calculations required:

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Mission statement

"To improve MuPAD/Sage as an extensible toolbox for computer exploration in combinatorics, and foster code sharing among researchers in this area"

Stratégie

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In a nutshell

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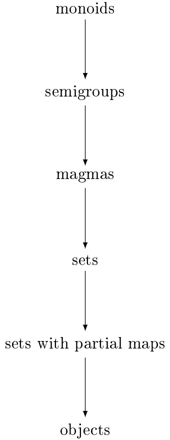
Managing large hierarchies of classes I

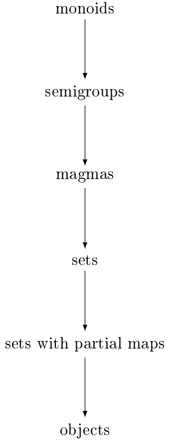


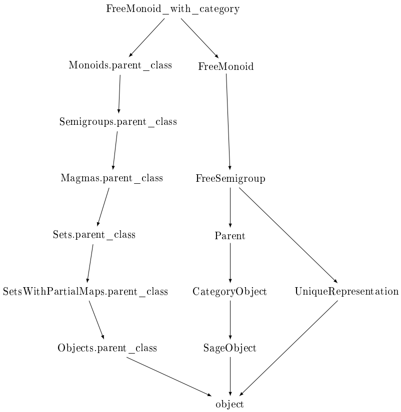
Categories

(and also: classes for tests, for morphisms)

Manual class hierarchies violate the “Single Point Of Truth” pattern

Managing large hierarchies of classes I





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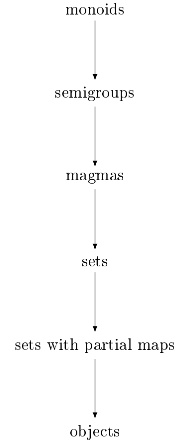

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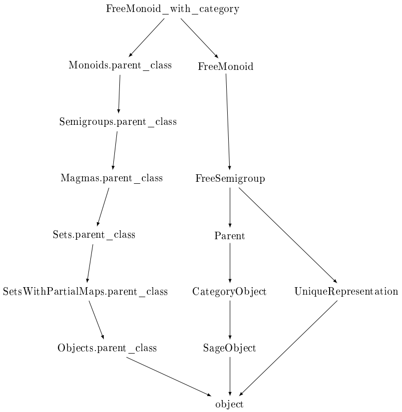

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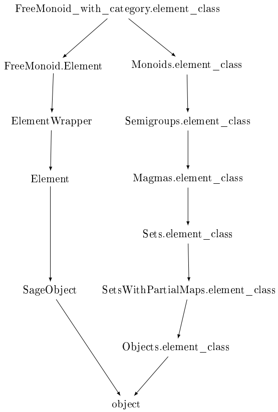

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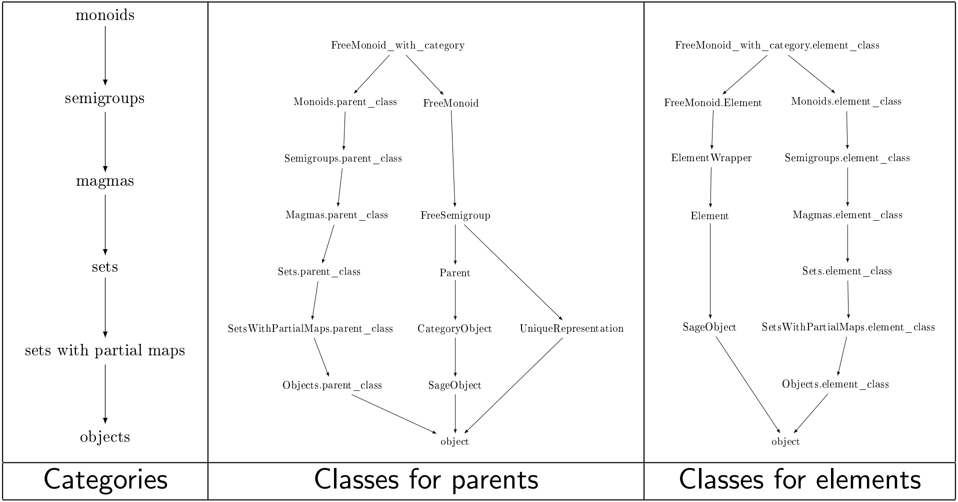

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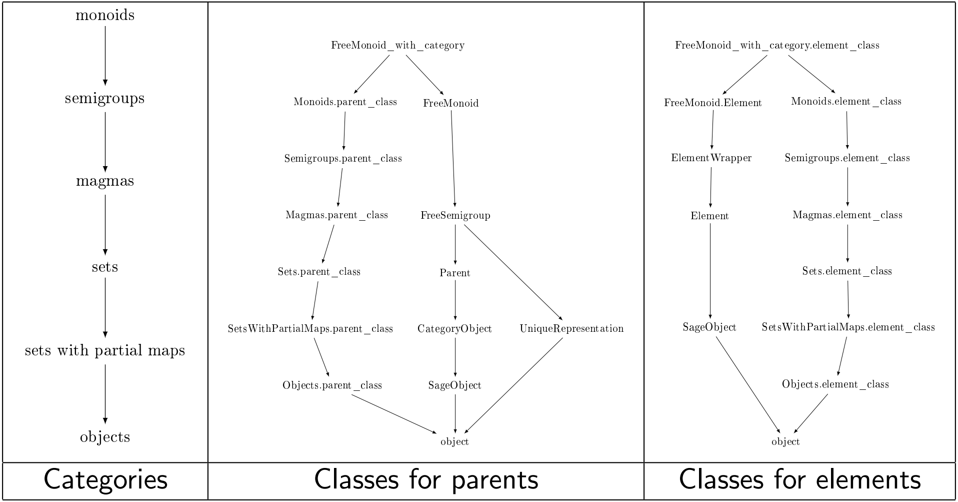

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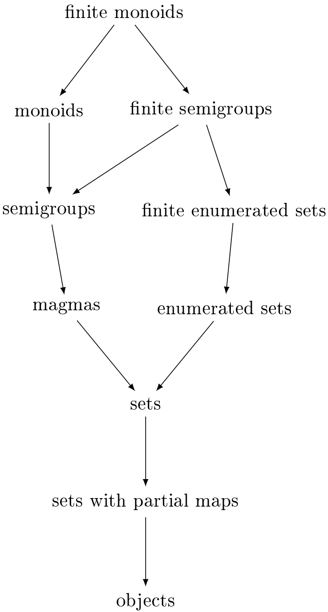

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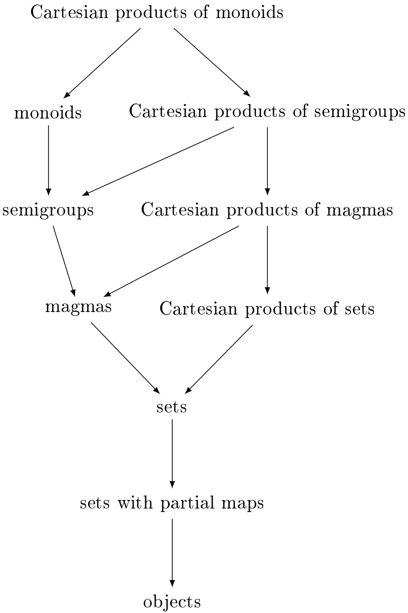

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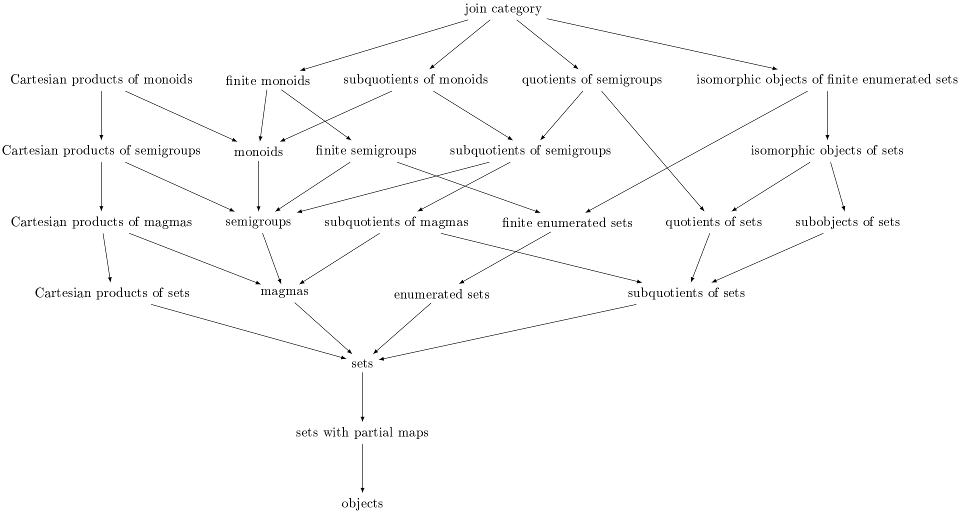

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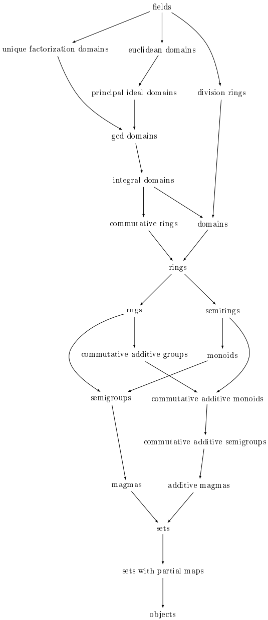

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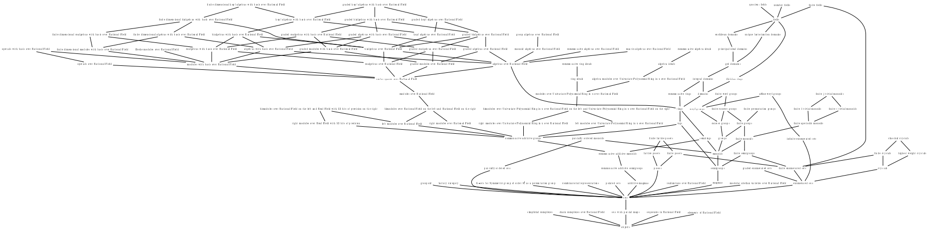

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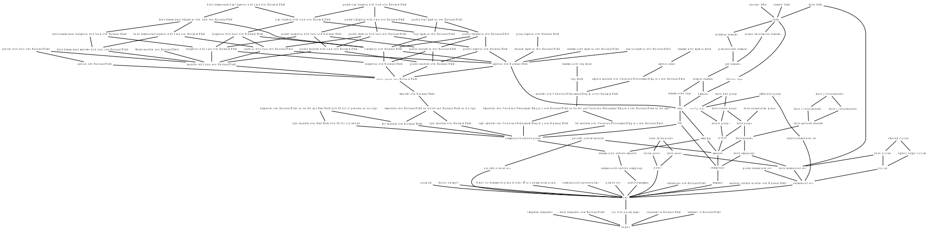
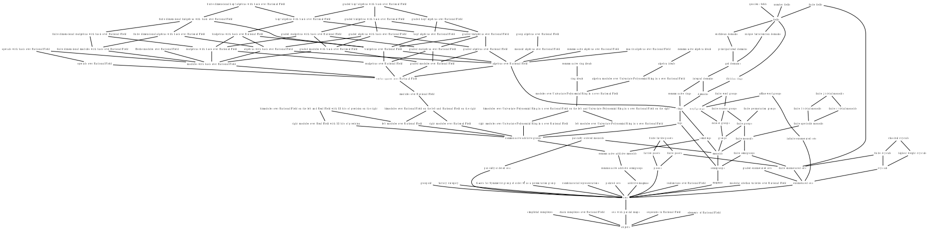


Problem

- *Control the* **combinatorial explosion**
- *Keep the size of the code and of the class hierarchy linear with the number of non trivial classes*

Managing large hierarchies of classes II



Solution

- Hierarchy of classes generated **dynamically** and **lazily**
- Lattice algorithmic to minimize the hierarchy

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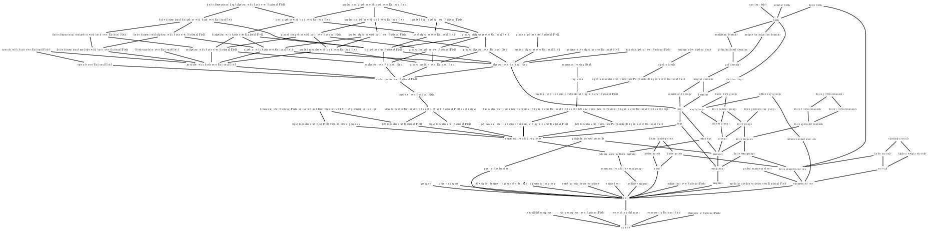

Challenges

- Refactoring of 400k lines / 10 referees / 1 year
- Compilation / partial compilation
- Serialization
- Documentation tools
- Dynamic class updates