

Report on Nicolas M. Thiéry,
Habilitation à diriger des recherches:
Algèbre combinatoire et effective des graphes aux
algèbres de Kac via l'exploration informatique

This thesis brings together very successfully a number of topics in algebraic combinatorics, together with other branches of mathematics such as group theory, theory of relations, and algebraic topology which serve as examples, sources of motivation. These fields also supply difficult test problems on which the methods can be tested.

The first chapter concerns commutative graded algebras. Such an algebra is associated with the age (the class of finite substructures, up to isomorphism) of a relational structure, and Maurice Pouzet has suggested that this is the right setting for Ulam's notorious *graph reconstruction problem* and its generalisations. Thiéry has proposed investigating these via computer algebra computations, specifically of SAGBI bases (analogues for subalgebra of Gröbner bases for ideals in polynomial rings). A nice conjecture on the relationship between spanning trees and spanning forests with two components may have applications in optimal design theory, where these things also arise. He moves on to consider ages of infinite structures, and proves a very important and long-sought theorem, showing that under a certain natural hypothesis the Hilbert series is a rational function. All the work exploits the computer to preform heroic computations, exploring the limits of what is possible computationally and hopefully pointing to a more theoretical approach.

The second chapter contains material which is both more technical and more mathematically central, on Hecke algebras. The emphasis is always on moving towards the concrete. For example, he has reformulated a famous conjecture of Wood in a form which can be attacked by the techniques of computer algebra for small values of n , and has indeed carried out such an attack. This is a lively area of algebraic combinatorics on which he has placed his mark.

The final chapter is a description of the computer algebra system **-Combinat* which he has developed with F. Hivert. The philosophy behind the system (which is not just another computer algebra system but stresses integration with existing packages and usability by researchers) is explained, and several impressive examples given of the system at work.

This is a very impressive thesis overall; it is full of ideas, open problems no less than results, and is the work of a researcher who can range over a large area of

mathematics (in algebraic combinatorics and computational algebra but also well beyond their boundaries) and has good judgment about what is important.

I strongly support him for the Habilitation.

Peter J. Cameron
November 18, 2008