L'algèbre et le monoïde de biHecke d'un groupe de Coxeter fini et leurs représentations

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arXiv:0804.3781v1 [math.RT]

Résumé

Dans cet exposé, nous présentons les résultats d'une série d'articles en collaboration avec Anne Schilling:
`arXiv:0711.1561`,`arXiv:0804.3781v3`,`arXiv:1012.1361`
Motivés par la théorie des représentations des algèbres de Hecke dégénérées et affine, ainsi que par le calcul de Schubert, nous associons à chaque groupe de Coxeter fini W une algèbre et un monoïde dits de biHecke. Leur construction s'appuie sur le modèle combinatoire usuel de la 0-algèbre de Hecke $H_0(W)$, i.e., pour le groupe symétrique, l'algèbre (ou le monoïde) engendré par les opérateurs de tri par bulles élémentaires. Plus précisément, ils sont obtenus comme algèbre ou monoïde engendrés conjointement par les opérateurs de tri et d'anti-tri.
Nous décrivons la théorie des représentations de l'algèbre de biHecke à l'aide de la combinatoire des descentes. De plus, nous montrons qu'il s'agit d'un quotient de l'algèbre de Hecke affine, expliquant par là les similarités de certaines représentations de cette dernière avec celles de $H_0(W)$. Finalement, la théorie des représentations du monoïde de biHecke étend celle de l'algèbre, grâce à une généralisation de la combinatoire des descentes relativement à tout intervale de W pour l'ordre faible.

Ces travaux s'appuient fortement sur l'exploration informatique. Nous illustrerons

notre démarche par quelques calculs typiques avec le logiciel Sage.

Schubert calculus, symmetric function

Divided differences operators:

$$\partial_i f(x_1, \dots, x_n) := \frac{f(\dots, x_i, x_{i+1}, \dots) - f(\dots, x_{i+1}, x_i, \dots)}{x_i - x_{i+1}}$$

$$\pi_i f := \partial_i(x_i f) \qquad \text{and} \qquad \hat{\pi}_i f := f - \pi_i f \qquad \dots$$

Problem

All these families satisfy the braids relations.
Describe the mixed relations ?

Applications:
appears in Iwahori-Hecke algebras, Schur symmetric functions, Schubert, Kazhdan-Lusztig polynomials, and Macdonald, (affine) Stanley symmetric functions, mathematical physics, Schur-Weyl duality for quantum groups, representations of $GL(\mathbb{F}_q)$, ...

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Representation theory: affine Hecke algebra

Theorem (Zelevinsky 1980)

Finite dim irrep of the affine Hecke algebra $\widetilde{\mathrm{H}}_n(q)$ are indexed by multisegments.

$$\widetilde{\mathrm{H}}_n(q) \simeq \mathrm{H}_n(q) \otimes \mathbb{C}[Y_1, \dots, Y_n]$$

$$Z(\widetilde{\mathrm{H}}_n(q)) = \mathbb{C}[Y_1, \dots, Y_n]^{\mathfrak{S}_{n+1}}$$

Principal specialization:

$$\mathcal{H}_n(q) := \widetilde{\mathrm{H}}_n(q) \,/\, \langle e_i(Y_1, \dots, Y_n) - e_i(1, q, \dots, q^{n-1}) \mid i = 1, \dots, n \rangle$$

- Bijection: Multisegments $\leftrightarrow$ subsets $I \subset \{1, \dots, n-1\}$.
- Base of irrep S_I indexed by descent classes of permutations.

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Definition (0-Hecke monoid $H_0(W)$ of a Coxeter group W)

Generators : $\langle \pi_1, \pi_2, \ldots \rangle$ (simple reflections)
Relations: $\pi_i^2 = \pi_i$ and braid relations

Theorem (Norton 1979-Carter 1986)

- *Simple modules S_I indexed by subsets $I \subset \{1, \ldots, n-1\}$.*
- *Base of indecomposable projective modules P_I indexed by descent classes of permutations.*

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Affine and 0 Hecke Algebras

- **Simple module** for the principal specialization $\mathcal{H}_n(q)$:
- **Indecomposable projective modules** for $H_0(W)$:

Combinatorics

- Indexed by subsets $I \subset \{1, \ldots, n-1\}$.
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Problem

Explain this coincidence of combinatorics

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Coxeter groups

Definition (Coxeter group W)

Generators : $(s_i)_{i\in S}$ (simple reflections)
Relations: $s_i^2 = 1$ and $\underbrace{s_i s_j \cdots}_{m_{i,j}} = \underbrace{s_j s_i \cdots}_{m_{i,j}}$, for $i \neq j$

Group algebra: $\mathbb{C}[W]$

Example (Type A_n: symmetric group $\mathfrak{S}_{n+1}$)

Generators: $(s_i)_{i=1,\ldots,n}$ (elementary transpositions)
Relations:

$$s_i^2 = 1 \qquad \text{for all } 1 \leq i \leq n,$$
$$s_i s_j = s_j s_i \qquad \text{for all } |i-j| > 1,$$
$$s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \qquad \text{for all } 1 \leq i \leq n-1.$$

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Definition (0-Hecke algebra $H(W)(0)$)

Generators : $(\pi_i)_{i\in S}$

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Basis: $(\pi_w)_{w\in W}$

Example (Type A_n)

Generators: $(\pi_i)_{i=1,\ldots,n}$ (adjacent comparators)

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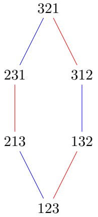
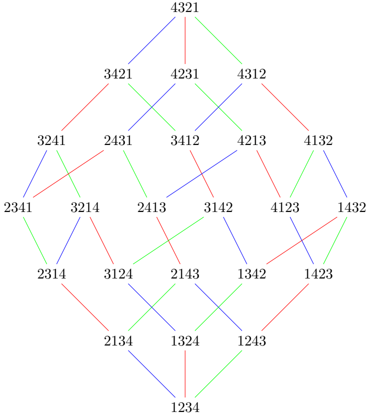
0-Hecke algebras and bubble sort

Example (Right regular representation for type A)

13854627	13854627	13854627
$\downarrow s_2$ $\downarrow s_6$	$\downarrow \pi_2$ $\downarrow \pi_6$	$\downarrow \overline{\pi}_2$ $\downarrow \overline{\pi}_6$
18354267	18354627	13854267
$\downarrow s_2$ $\downarrow s_6$	$\downarrow \pi_2$ $\downarrow \pi_6$	$\downarrow \overline{\pi}_2$ $\downarrow \overline{\pi}_6$
13854627	18354627	13854267


321
231
312
213
132
123


4321
3421
4231
4312
3241
2431
3412
4213
4132
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3214
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4123
1432
2314
3124
2143
1342
1423
2134
1324
1243
1234

Hecke algebras

Take q_1 and q_2 parameters, and set $q := -\frac{q_1}{q_2}$.

Definition (Hecke algebra $\mathrm{H}(W)(q_1, q_2)$)

Generators : $(T_i)_{i \in S}$ Relations: $(T_i - q_1)(T_i - q_2) = 0$ and $\underbrace{T_i T_j \cdots}_{m_{i,j}} = \underbrace{T_j T_i \cdots}_{m_{i,j}}$, for $i \neq j$

Basis: $(T_w)_{w \in W}$

- At $q = 1$: group algebra $\mathbb{C}[W]$
- At $q = 0$: 0-Hecke algebra $\mathrm{H}(W)(0)$
- At $q_1 = q_2 = 0$: nilCoxeter algebra
- At q not 0 nor a root of unity: isomorphic to $\mathbb{C}[W]$

Realization of T_i as operator in $\mathrm{End}(\mathbb{C}W)$:

$$T_i := (q_1 + q_2)\pi_i - q_1 s_i$$

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A silly idea during a brainstorm (Thibon, Novelli, H., T., 2003)

Definition (BiHecke algebra HW of a Coxeter group W)

Glue $\mathbb{C}[W]$ and $H(W)(0)$ on their right regular representations:

$$HW := \langle (\pi_i, s_i)_{i \in S} \rangle \quad \subset \quad \mathrm{End}(\mathbb{C}W)$$

- Any interesting structure?
- Contains all Hecke algebras by construction
- Type A: dimension and dimension of the radical in the Sloane!

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The BiHecke algebra of rank 1

$$W := \{1, s\} \qquad \mathbb{C}W := \mathbb{C}.1 \oplus \mathbb{C}.s$$

$$\mathrm{id} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad s = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \pi = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad \overline{\pi} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Relations and some natural bases of HW

$$s\pi = \pi, \qquad \pi s = \overline{\pi}, \qquad \overline{\pi} + \pi = 1 + s$$

$$\{\mathrm{id}, s, \pi\} \qquad \text{or} \qquad \{\mathrm{id}, \pi, \overline{\pi}\}$$

Dimension 1 simple and projective modules

$$(1-s).\mathrm{id} = (1-s), \qquad (1-s).s = -(1-s), \qquad (1-s).\pi = 0$$

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Structure of BiHecke algebras

Theorem (H.,T., 2005)

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- HW^* *algebra of left symmetry preserving operators*
- *Basis of* HW*:* $\{w\pi_{w'} \mid \mathrm{D}_R(w) \cap \mathrm{D}_L(w') = \emptyset\}$
- *Rep. theory governed by the combinatorics of descents*
- HW *Morita equivalent to the poset algebra of boolean lattice*
- *Projective & simple modules indexed by parabolic subgroups*
  Restriction of simple:
  - *Exactly the Young's ribbon representation of* W
  - *Exactly the projective modules of* $\mathrm{H}(W)(0)$

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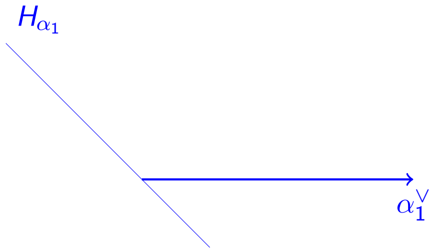
Is there a link with the affine Hecke algebra?

Level 0 action of the type A_1^1 affine Hecke algebra



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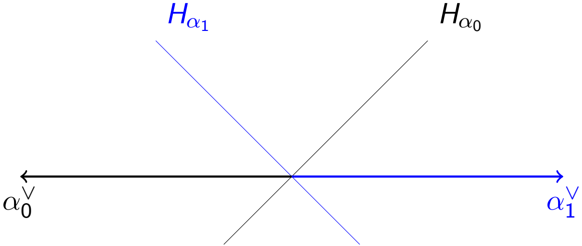

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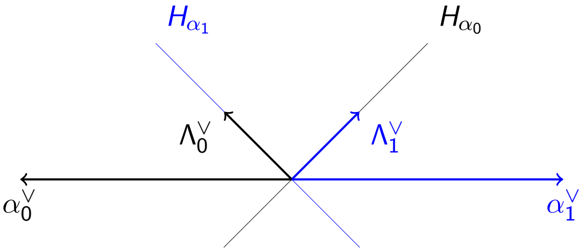

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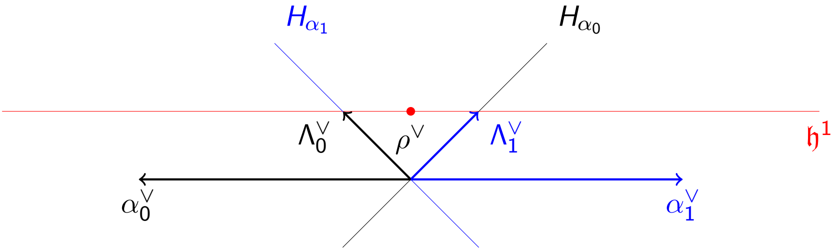

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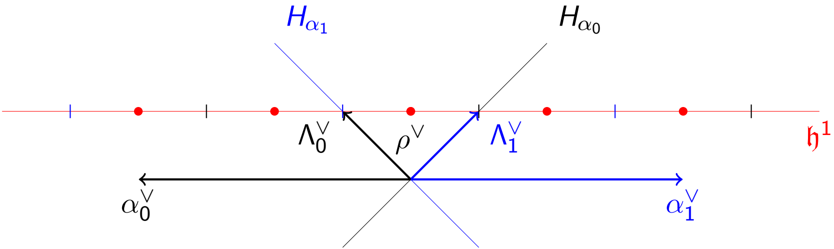

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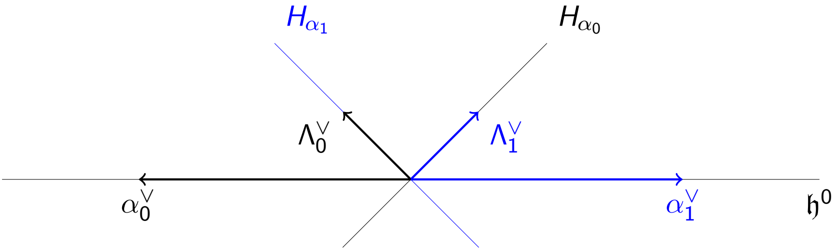

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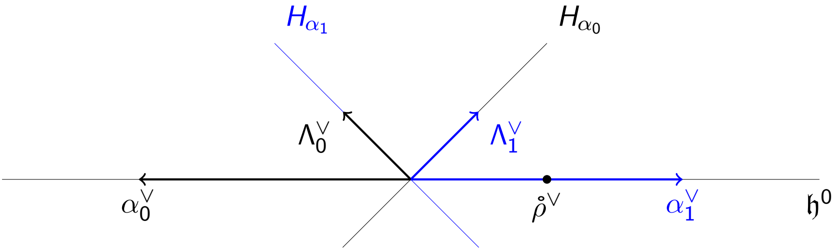

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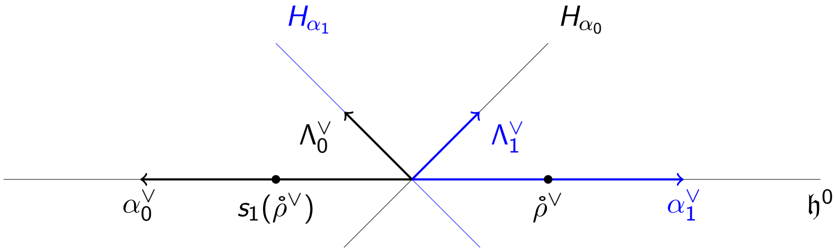

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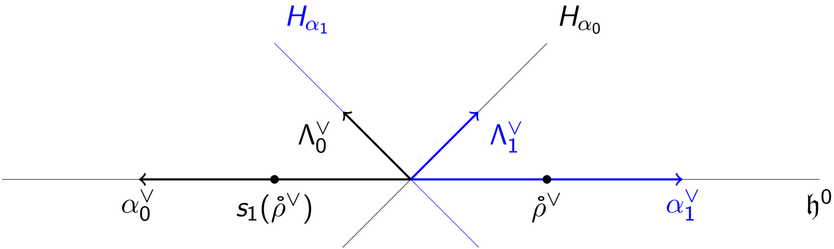

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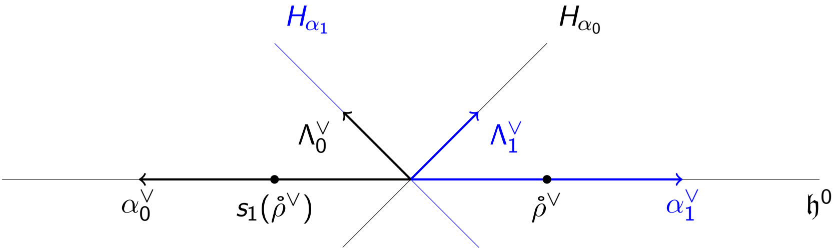

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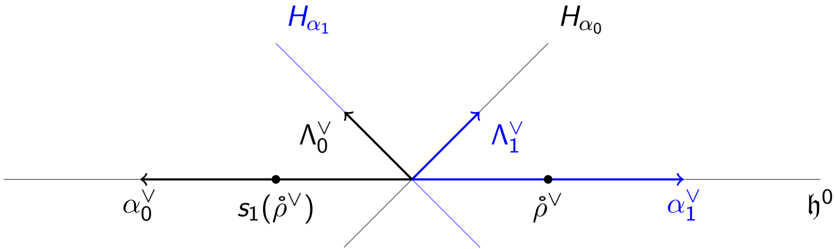

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Theorem (H.,S.,T. 2008)

W: affine Weyl group
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1243

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1423

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4123

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4123

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4132

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4312

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4321

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Underlying combinatorics: right permutohedron

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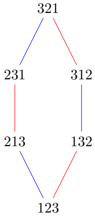
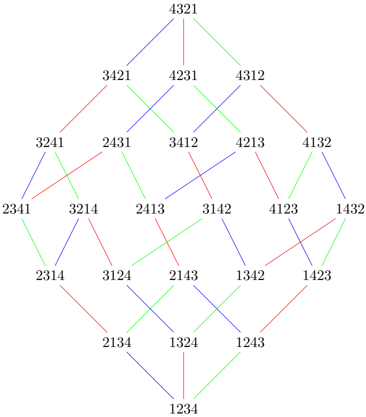
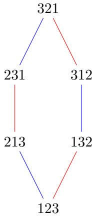


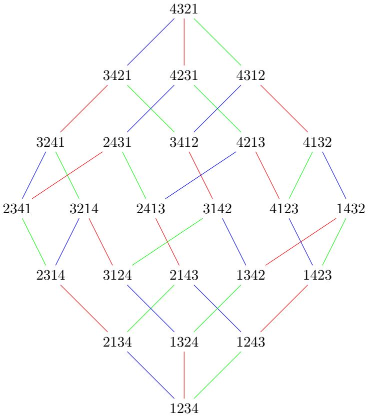


Type A: Bubble (anti) sort algorithm

4321

Underlying combinatorics: right permutohedron





Elementary transpositions: $s_1, s_2, s_3, \ldots$

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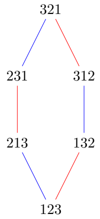

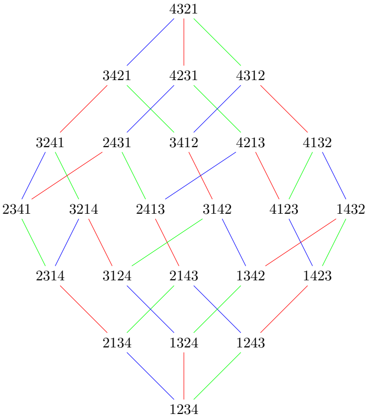

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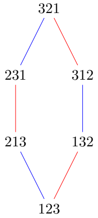
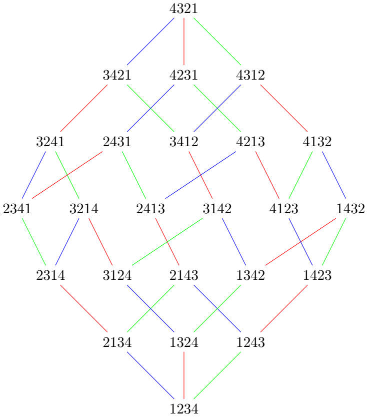
Elementary bubble antisort operators: $\pi_1, \pi_2, \pi_3, \ldots$

Type A: Cyclic bubble sort

$\pi_1, \pi_2, \pi_3, \ldots$ antisort 12345 into 54321 There is no return!





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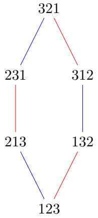

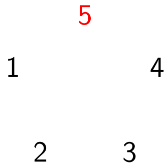

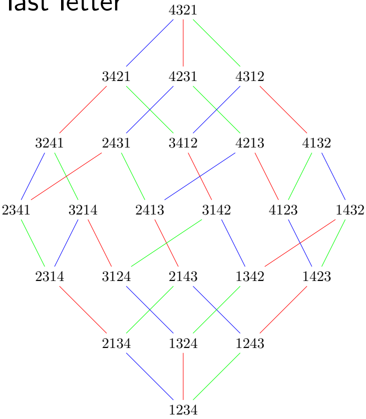

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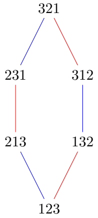

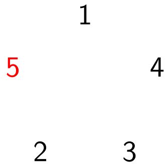

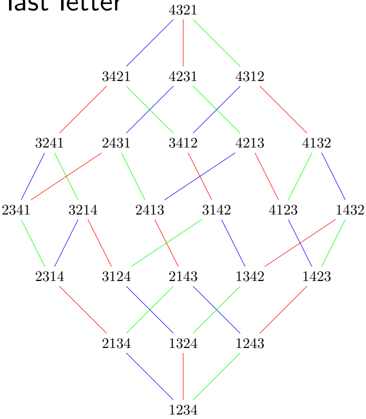

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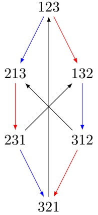

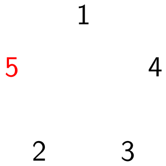

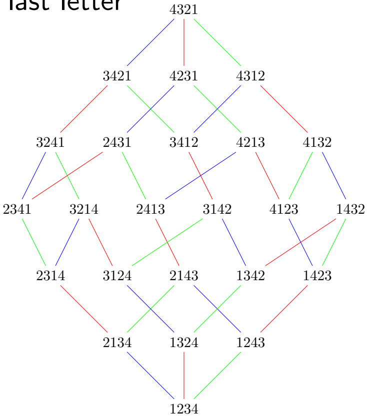

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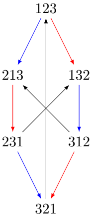

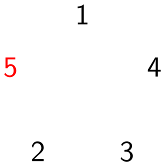

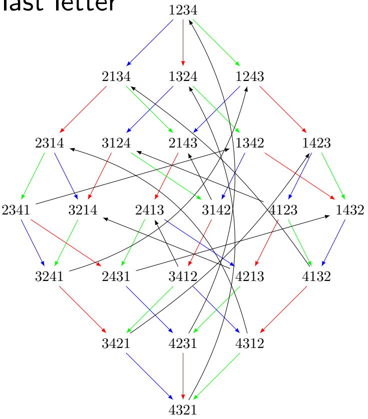

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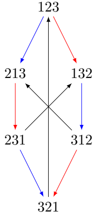

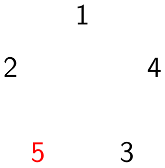

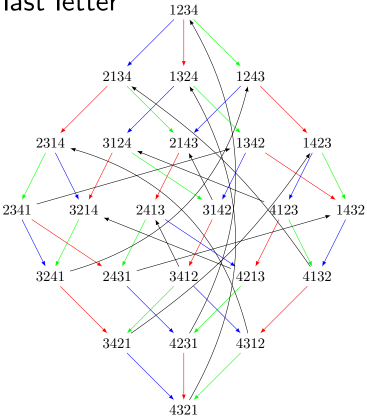

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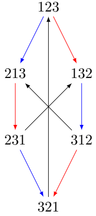

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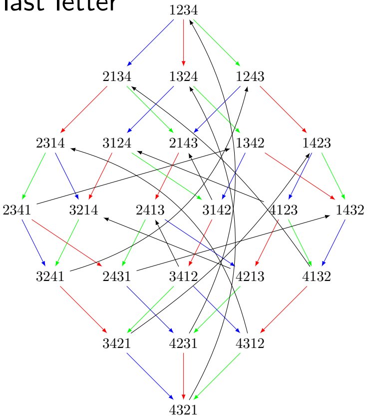

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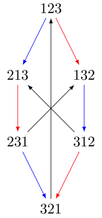

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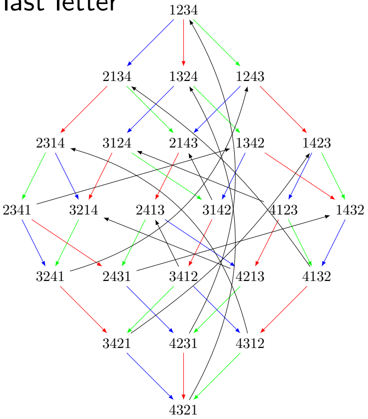

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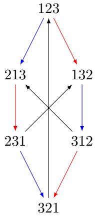

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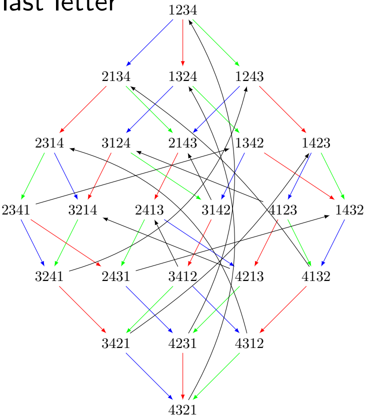

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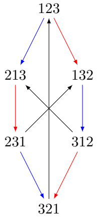

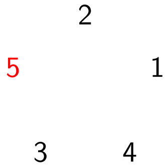

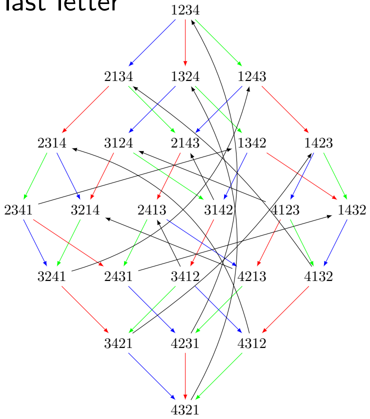

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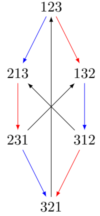

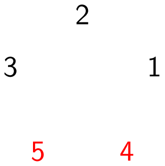

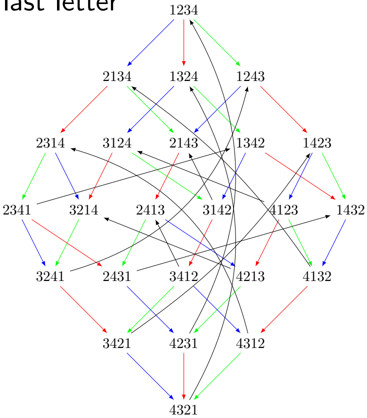

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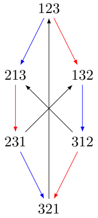

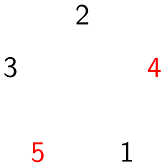

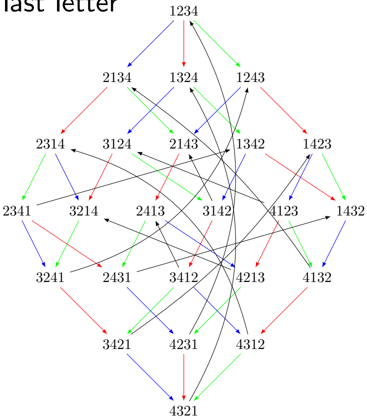

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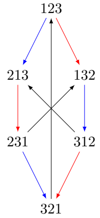

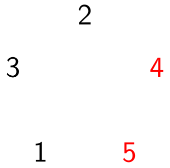

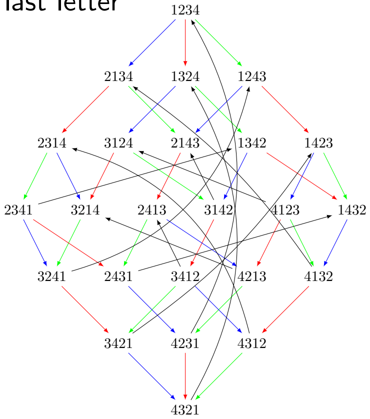

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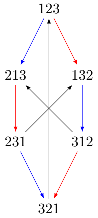

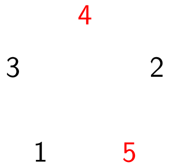

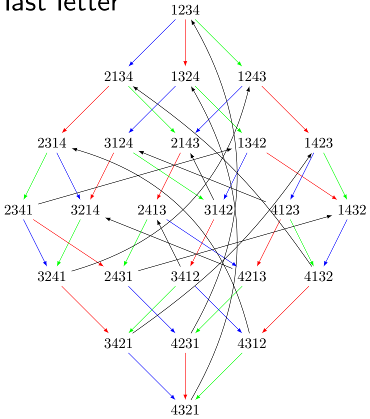

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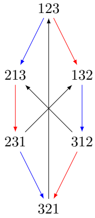

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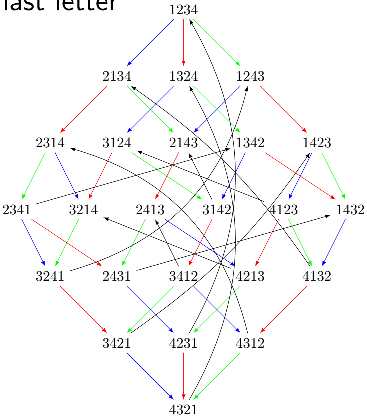

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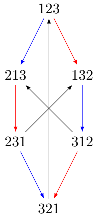

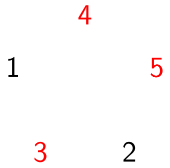

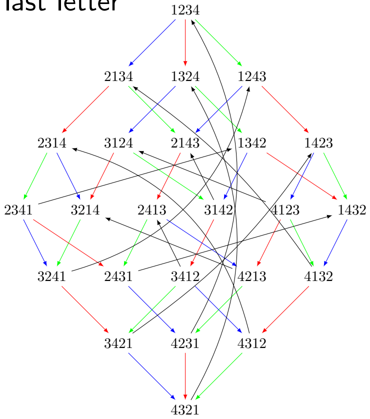

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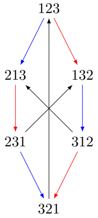

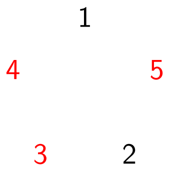

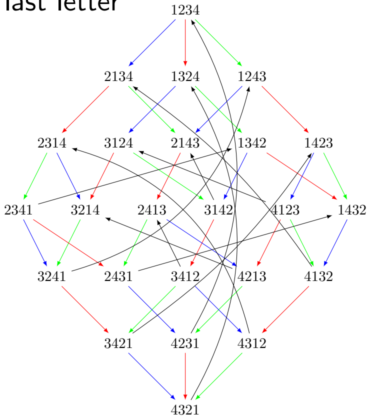

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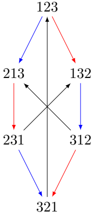

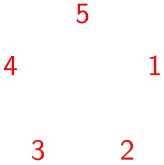

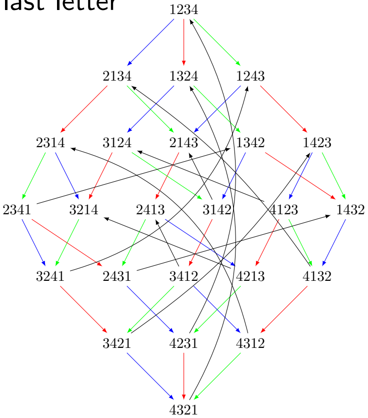

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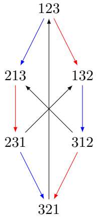

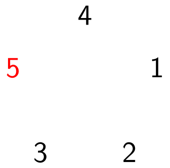

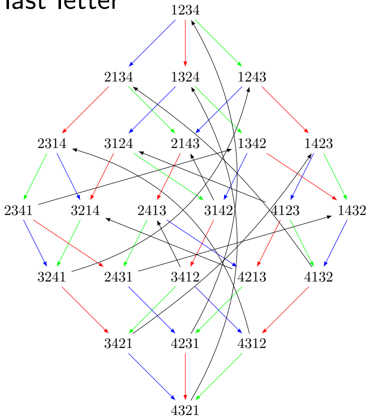

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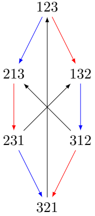

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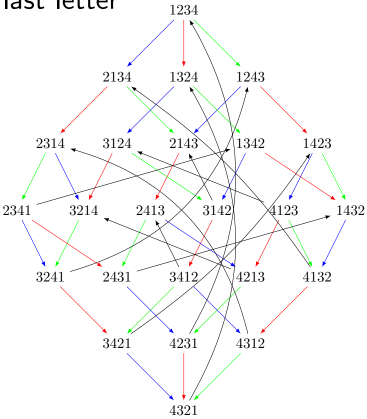

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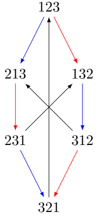

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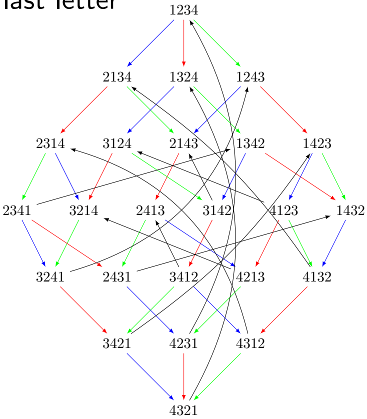

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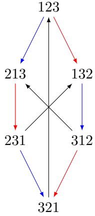

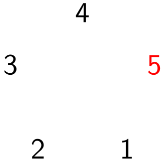

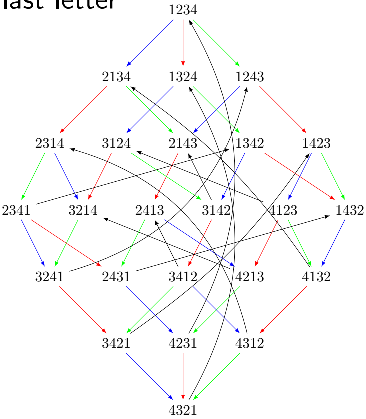

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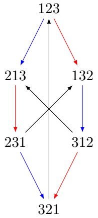

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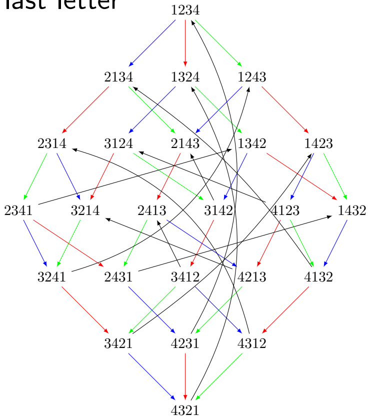

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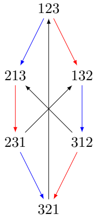

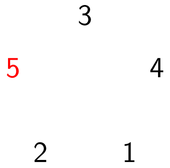

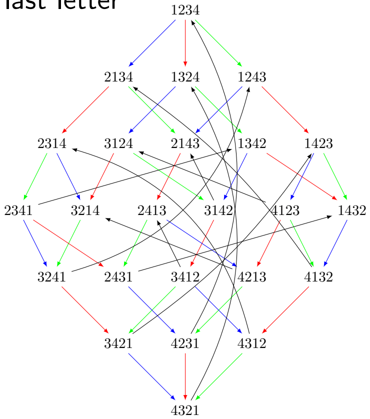

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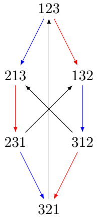

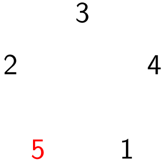

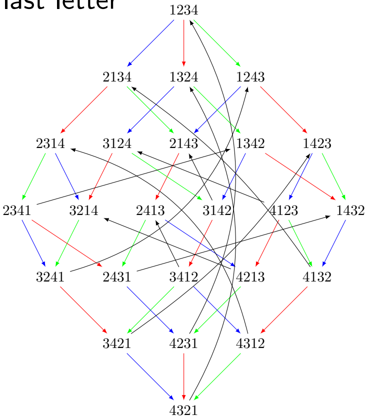

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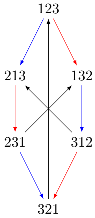

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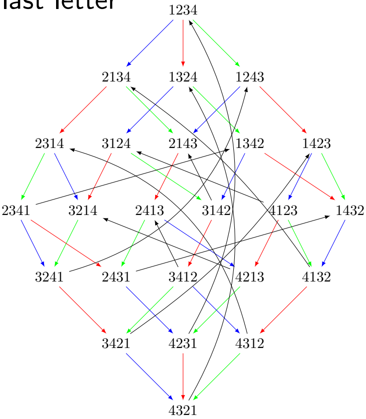

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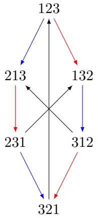

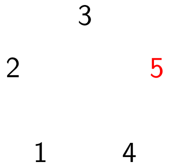

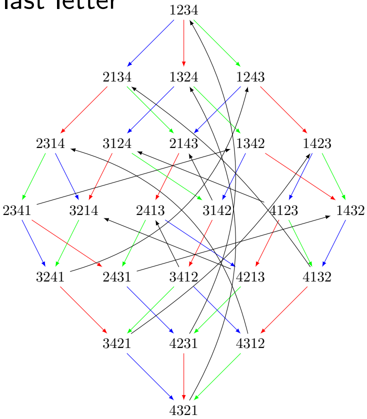

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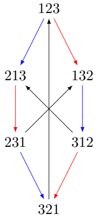

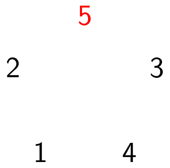

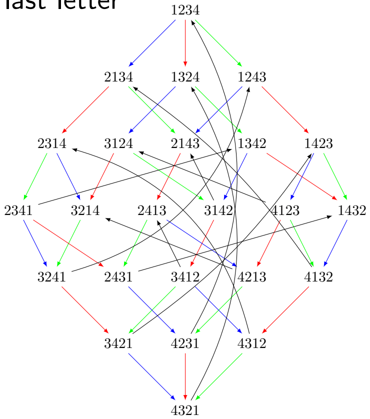

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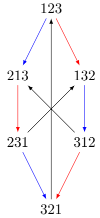

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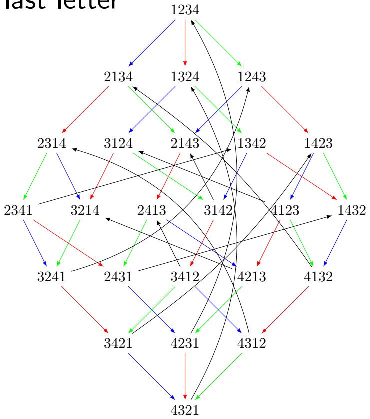

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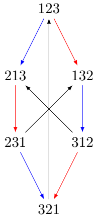

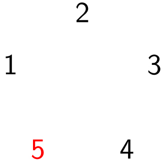

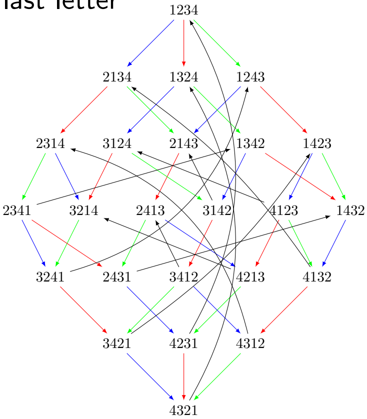

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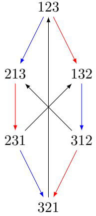

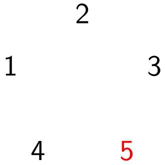

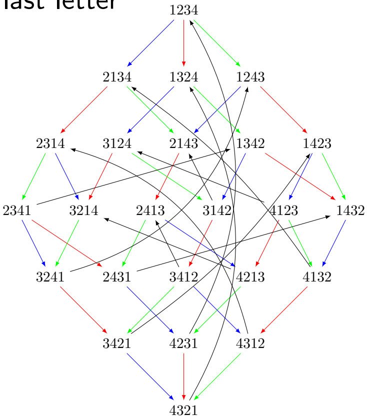

Proposition (HT'08)

$\pi_0, \pi_1, \pi_2, \pi_3, \ldots$ *act transitively on permutations*

Type A: Cyclic bubble sort

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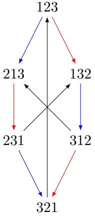

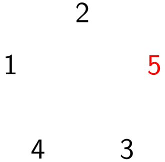

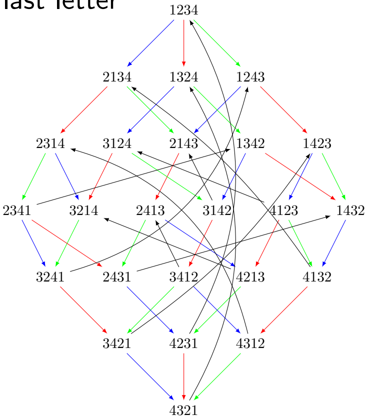

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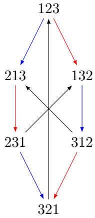

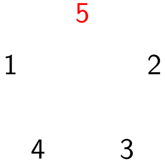

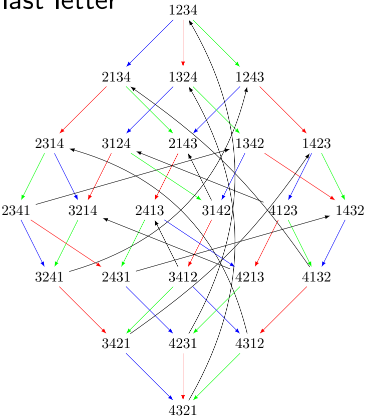

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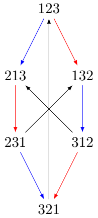

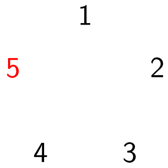

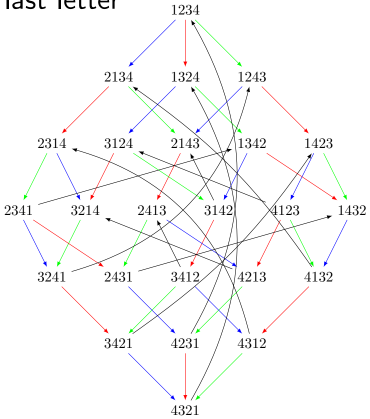

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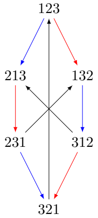

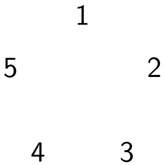

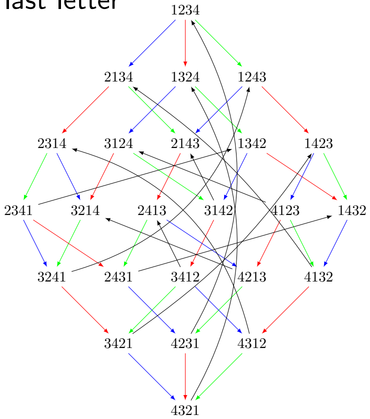

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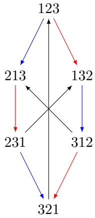

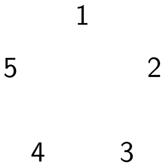

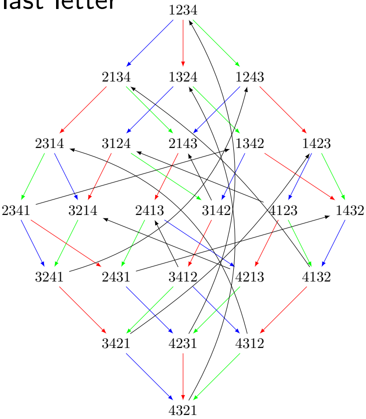

Proposition (HT'08)

$\pi_0, \pi_1, \pi_2, \pi_3, \ldots$ *act transitively on permutations*

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types* B, C, D
- *Existence for all types (including twisted)*

Proof

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$$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$$

$$\underline{1234}$$

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$\underline{12}\underline{34}$

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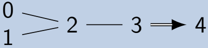

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

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$2\underline{3}1\underline{4}$

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$2\underline{34}1$

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$\underline{3}2\underline{4}1$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{34}21$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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- *Existence for all types (including twisted)*

Proof

- Type B:

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$\underline{34}21$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

4321

- Type-free induction strategy
- Case by case induction step
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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$432\underline{1}$

- Type-free induction strategy
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Recursive sorting algorithms in other types

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- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$43\underline{1}2$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$4\underline{1}32$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{1}432$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{1}43$2

- Type-free induction strategy
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Recursive sorting algorithms in other types

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- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

 $1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{1}43\underline{2}$$

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Recursive sorting algorithms in other types

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Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{1}4\underline{2}3$

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Recursive sorting algorithms in other types

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Proof

- Type B:  $1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{1}\underline{2}43$$

- Type-free induction strategy
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B:

$$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$$

$$\underline{12}43$$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{1}\underline{2}4\underline{3}$$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: $\begin{smallmatrix}0\\1\end{smallmatrix} > 2 - 3 \Rightarrow 4$ $\quad 1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$$\underline{1}\underline{2}\underline{3}4$$

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Recursive sorting algorithms in other types

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- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{123}4$

- Type-free induction strategy
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
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Proof

- Type B:

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$21\underline{3}4$

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Recursive sorting algorithms in other types

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$21\underline{3}4$

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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$2\underline{3}14$

- Type-free induction strategy
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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$2\underline{3}14$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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- *Existence for all types (including twisted)*

Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}214$

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}214$

- Type-free induction strategy
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}241$

- Type-free induction strategy
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}24\underline{1}$

- Type-free induction strategy
- Case by case induction step
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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
- *Existence for all types (including twisted)*

Proof

- Type B: 

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}2\underline{1}4$

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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$\underline{3}\underline{1}24$

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Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

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Proof

- Type B: 

  $1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

  $\underline{31}24$

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1324

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Recursive sorting algorithms in other types

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1342

- Type-free induction strategy
- Case by case induction step
  Brute force on computer for the exceptional types (E_8!)

Recursive sorting algorithms in other types

Proposition (S.,T.,2007)

- *Similar algorithms for types B, C, D*
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Proof

- Type B:

$1 < 2 < 3 < 4 < \underline{4} < \underline{3} < \underline{2} < \underline{1}$

$134\underline{2}$

- Type-free induction strategy
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- Type B: 

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$13\underline{2}4$

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$1\underline{2}34$

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$\underline{2}134$

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Proposition (S.,T.,2007)

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- *Existence for all types (including twisted)*

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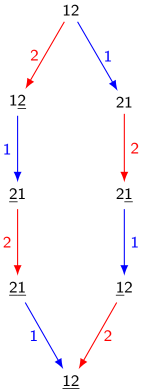

π_1, π_2 on C_2

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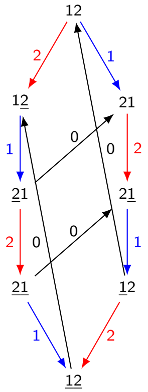

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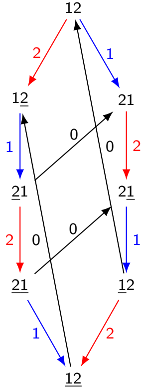

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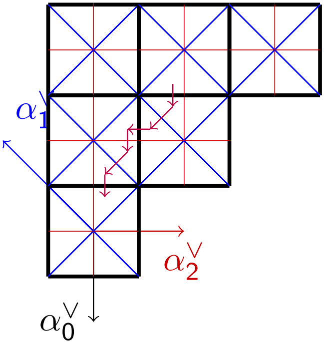

Alcove picture at level 1

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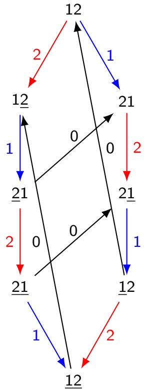

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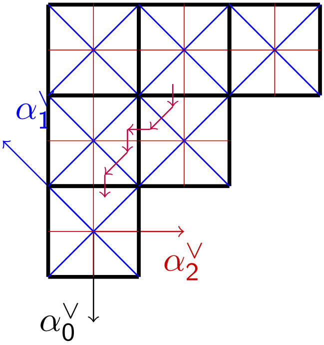

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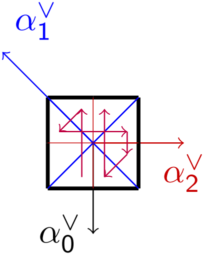

Quotient at level 0
(Steinberg torus)

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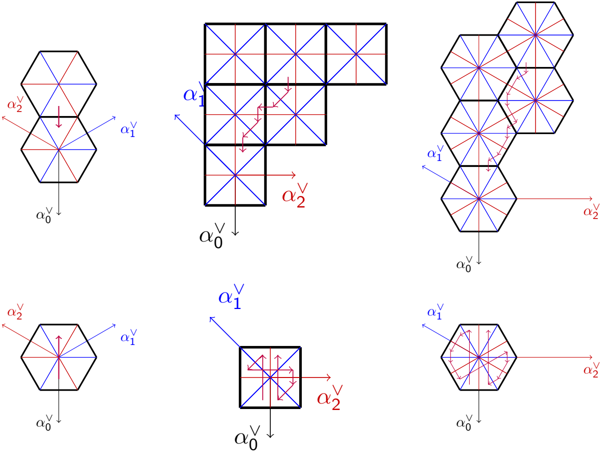

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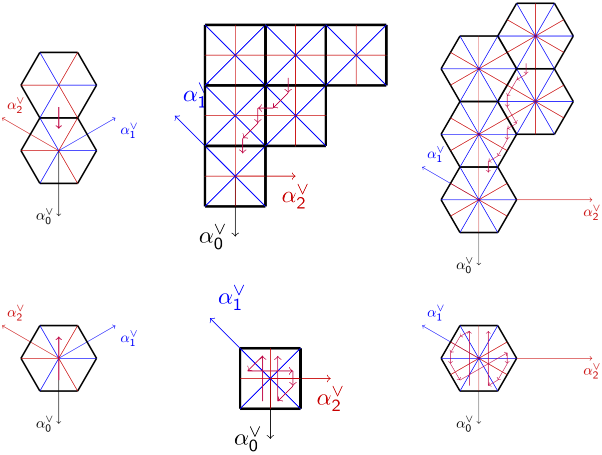

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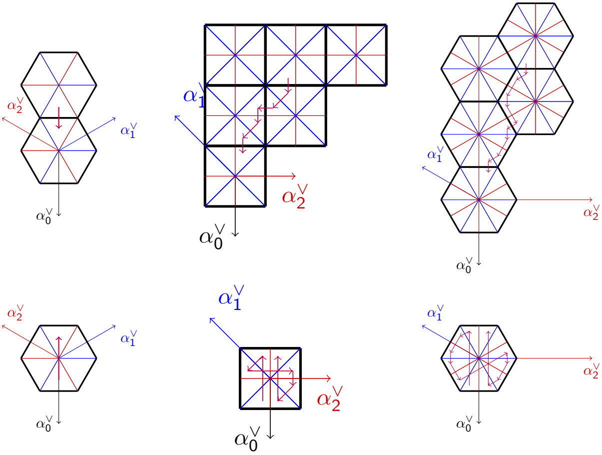

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BiHecke algebras and principal series representations

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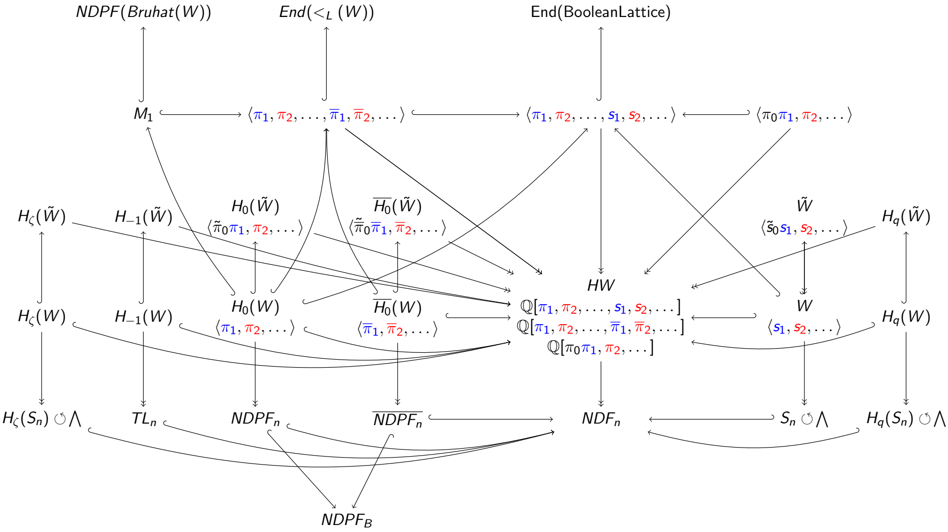
Partial conclusion

- BiHecke algebras:
  - Many equivalent definitions
  - Nice structure and representation theory
  - (type A) Connections with NCSF, parking functions
  - Connections with 0-Hecke and affine Hecke algebras
- Where does this structure come from?
- Is it useful?

The Big Picture



The bi-Hecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \dots, \overline{\pi}_1, \overline{\pi}_2, \dots \rangle$

$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

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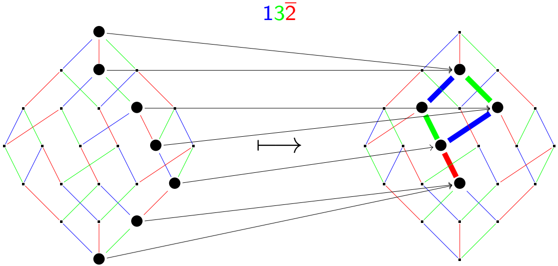
Key combinatorial lemma



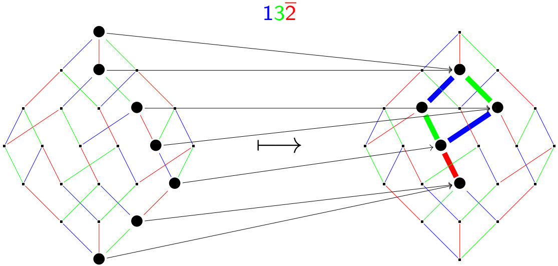
Lemma

For $f \in M(W)$ and $w \in W$: $(s_i w).f = w.f$ or $s_i\,(w.f)$

Proof.

Exchange property / associativity □

Key combinatorial lemma



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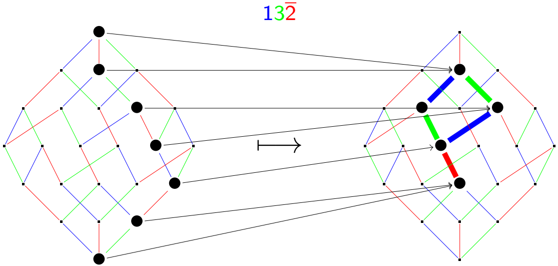

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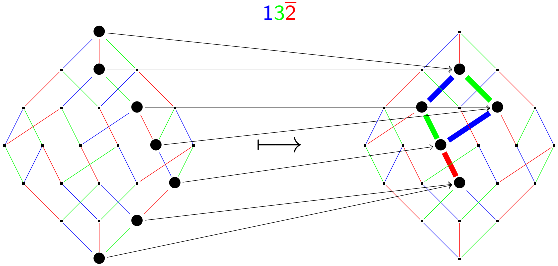
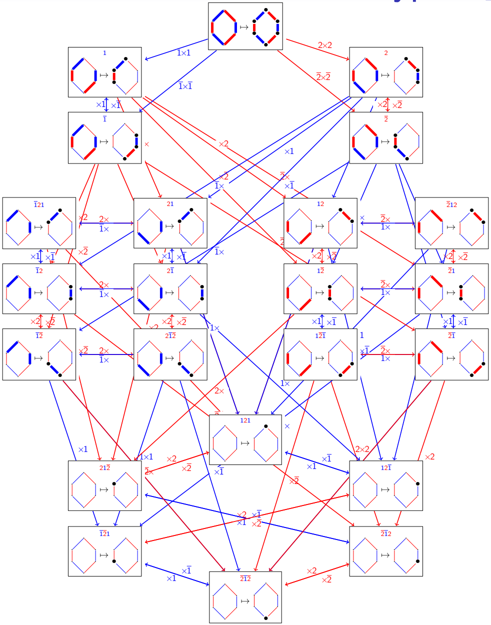


Corollary

- *If* $w = uv$*, then* $(uv).f = u' \, (v.f)$*, where* $u' <_B u$
- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

The biHecke monoid for type A_2



Representation theory of $M(W)$

Theorem (HST'08)

$M(W)$ admits $|W|$ simple modules

Sketch of proof.

- M acts transitively on intervals $[u, v]_L$
- The image set of an idempotent is an interval $[u, v]_L$
- $\exists!$ e_w idempotent with image set $[1, w]_L$, for any $w \in W$
- $(e_w)_{w \in W}$: transversal of the regular J-classes
    - $f = uev$ if and only if $\mathrm{im}(f)$ is a subinterval of $\mathrm{im}(e)$

□

Problem

Dimension of simple and projective modules?

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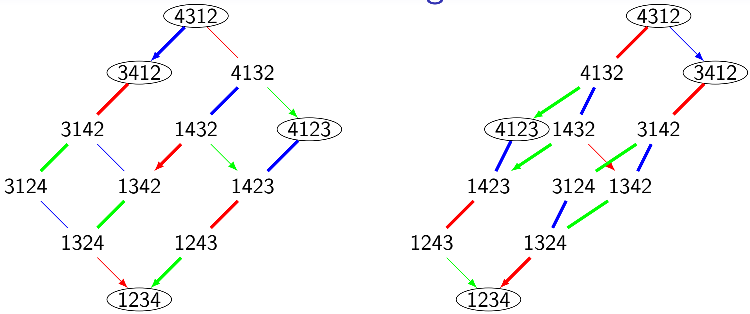
Translation algebras



Definition (Translation algebra)

$T_w := \mathbb{Q}[\pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\} \quad \Longrightarrow \quad$ Submodules P_J
- T_w: max. algebra stabilizing all $P_J \quad \Longrightarrow \quad$ Repr. theory
- T_w quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

Conclusion

General strategy:

- Find combinatorial models for algebras and representations
- As simple as possible, but no simpler
- Concrete and effective
- Find the right point of view where proofs become trivial
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Mission

« Améliorer MuPAD/Sage comme boîte à outils extensible pour l'exploration informatique en combinatoire algébrique, en fédérant et mutualisant les efforts de développements des chercheurs »

Stratégie

- Licence libre pour partager avec le plus grand nombre
  En restant pragmatique dans les collaborations
- Développement décentralisé et international
  Garantie d'indépendance vis-à-vis des tutelles
- Développé par des chercheurs pour des chercheurs
  Avec un usage plus large en vue
- Coeur du développement par des permanents
  Les doctorants se concentrent sur leurs propres besoins
- Chaque ligne de code justifiée par un projet de recherche
  Avec une vision à long terme (développement agile)
- Inspiration des informaticiens:
  Concepts et méthodologies de programmation
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Le projet Sage-Combinat: 10 ans après

En quelques chiffres

- MuPAD/C++: 600 classes, 5000 méthodes, 160k lignes de code+doc+tests
- Sage (Python): 300 tickets / 200k lignes de code+doc+tests
- Financements: ANR, PEPS, NSF, Google Summer of Code
- **70+ articles de recherche:**
  - Combinatoire algébrique, énumérative, des mots, ...
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Une communauté internationale

- Barcelonne, Davis, Lyon, Marne, Marseille, Montpellier, Orsay, Paris, Rouen, Philadelphie, Seattle, Stanford, Montreal, Toronto, ...
- Abbad, Berg, Borie, Bump, Bandlow, Boussicault, Chapoton, Delecroix, Dehaye, Denton, Descouens, Drake, Gomez Diaz, Feray, Hansen, Hemmecke, Hivert, Jones, Labbé, Laigle-Chapuy, Laugerotte, Lemeur, Mathas, Molinero, Monteil, Musiker, Novelli, Nzeutchap, Pon, Rubey, Saliola, Schilling, Shimozono, Stump, Tevlin, Thiéry, Walker, Wang, Zabrocki, ...

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